Layered Mechanisms

Game Theory · Module 10 — Strategic Systems Integration

Node 10.2

Position in Knowledge Graph

System: Game Theory **Structural Domain:** Strategic Systems Integration **Node:** 10.2

Module 6 introduced mechanism design as a problem of choosing the rules of a game to achieve a desired outcome. 10.1 showed that every game sits inside an institution. 10.2 brings these together: in most real-world settings, a mechanism designer does not operate in a vacuum but within an **outer mechanism** imposed by a regulator, a market structure, or a set of social norms. The result is **mechanism design within mechanism design** — a layered structure where the inner mechanism is optimized subject to the constraints imposed by the outer mechanism, and the outer mechanism is chosen anticipating the inner mechanism's behavior. This layered structure is ubiquitous: auctions run inside regulated markets, poker tournaments run inside casino licensing regimes, corporate governance structures sit inside securities law, and electoral systems sit inside constitutional frameworks. 10.2 formalizes the nesting, characterizes optimality at each layer, and identifies the new phenomena that arise from the interaction between layers.

Formal Definition

A **layered mechanism** is a pair $(M^{\text{out}}, M^{\text{in}})$ where:

- $M^{\mathbf{out}} = (N^{\mathbf{out}}, \Sigma^{\mathbf{out}}, g^{\mathbf{out}})$ is the **outer mechanism** (regulator, platform, constitution). Here N^{out} is the set of outer-level agents (regulators, platform designers), Σ^{out} is their message space, and $g^{\text{out}} : \Sigma^{\text{out}} \to \mathcal{M}$ maps outer messages to a set of **permitted inner mechanisms** $\mathcal{M}$.
- $M^{\mathbf{in}} = (N^{\mathbf{in}}, \Sigma^{\mathbf{in}}, g^{\mathbf{in}})$ is the **inner mechanism** (auction, poker game, contract), with $M^{\text{in}} \in \mathcal{M}$ constrained by the outer mechanism's output.

The **sequential structure** is:

1. Outer-level agents choose $\sigma^{\text{out}} \in \Sigma^{\text{out}}$, producing the regulatory framework $g^{\text{out}}(\sigma^{\text{out}}) = \mathcal{M}' \subseteq \mathcal{M}$.
2. Inner-level designer chooses $M^{\text{in}} \in \mathcal{M}'$ to maximize their objective subject to outer constraints.
3. Inner-level agents play the game induced by M^{in}.

The **layered equilibrium** is a triple $(\sigma^{*\text{out}}, M^{*\text{in}}, \sigma^{*\text{in}})$ where:

$$\sigma^{*\text{in}} \in \text{BNE}(M^{*\text{in}}), \qquad M^{*\text{in}} \in \arg\max_{M \in \mathcal{M}'} V^{\text{in}}(M, \sigma^{*\text{in}}(M)), \qquad \sigma^{*\text{out}} \in \arg\max V^{\text{out}}(\mathcal{M}', M^{*\text{in}}(\mathcal{M}'), \sigma^{*\text{in}})$$

where V^{in} and V^{out} are the inner designer's and outer designer's objective functions, and each level optimizes taking the lower levels' equilibrium behavior as given. This is a **three-level principal-agent problem**: the outer principal (regulator) designs constraints, the inner principal (mechanism designer) designs the mechanism, and the agents play.

Intuition

Consider an auction house that sells spectrum licenses under government regulation. The government (outer mechanism) sets rules: reserve prices, anti-collusion provisions, eligibility criteria, spectrum caps. The auction house (inner mechanism) designs the specific auction format — sealed-bid, ascending, combinatorial — subject to these constraints. The bidders (agents) play the resulting auction game.

Neither layer alone captures the full strategic picture. Analyzing the auction in isolation misses the regulatory constraints that restrict the auction designer's choices. Analyzing the regulation in isolation misses the strategic behavior of the auction designer who optimizes within the regulatory framework. The layered structure is essential to understanding the outcome.

The same nesting appears everywhere:

- **Poker:** The casino (outer mechanism) sets rake, buy-in caps, game-selection rules, and enforcement. The tournament director (inner mechanism) designs blind structures, payout tables, and ICM-relevant rules. The players play within both layers.
- **Markets:** The regulator (outer) sets disclosure requirements, capital adequacy rules, and anti-manipulation provisions. The exchange (inner) designs the order book, matching algorithm, and fee structure. Traders trade.
- **Organizations:** The legal system (outer) sets employment law, fiduciary duties, and tax rules. The firm (inner) designs incentive contracts, reporting structures, and promotion criteria. Employees work.

The key phenomenon of layered mechanisms is **strategic propagation between layers**. A change at the outer layer propagates through the inner designer's optimization to the agents' equilibrium behavior. Conversely, information about agent behavior propagates upward through the inner designer's reports (or regulatory audits) to the outer layer. This bidirectional flow means that optimizing at one layer in isolation is typically suboptimal: the outer designer must anticipate the inner designer's strategic response, and the inner designer must anticipate the agents' strategic response. This is the **nested principal-agent problem** that 10.3 will specialize to bilateral contracts.

Structural Role

10.2 occupies the structural position between 10.1 (the general concept of institutional embedding) and 10.3 (the specific bilateral case of contracts). It is the module's theory of mechanism-design interaction:

- **From 6.1–6.3:** Mechanism design gave us the tools for a single layer — VCG, revenue equivalence, optimal auctions. 10.2 shows what happens when you stack these tools.
- **To 10.3:** Contracts are the simplest layered mechanism — an outer legal system constraining an inner bilateral agreement. The principal-agent theory of 10.3 is a special case of the layered formalism.
- **To 10.6:** Strategic ecosystems are the limit of many interacting layers. 10.2 provides the two-layer case that the ecosystem generalizes.

The structural novelty of 10.2 is that **the inner mechanism's optimality is constrained optimality**, and the binding constraints are themselves the output of a strategic process. This means that the "optimal mechanism" at the inner layer depends on the outer mechanism, and the "optimal regulation" at the outer layer depends on the inner mechanism's response. The fixed-point nature of this interaction is what makes layered mechanism design harder than single-layer design.

Mathematical Development

The constrained mechanism-design problem

The inner designer solves:

$$\max_{M^{\text{in}} \in \mathcal{M}'} V^{\text{in}}(M^{\text{in}}, \sigma^*(M^{\text{in}}))$$

subject to: - $M^{\text{in}} \in \mathcal{M}'$ (outer-mechanism constraint). - $\sigma^*(M^{\text{in}}) \in \text{BNE}(M^{\text{in}})$ (incentive compatibility at the agent level). - Participation constraints: agents must voluntarily participate.

Without the outer constraint, this is the standard Myerson optimal auction problem (6.3). With the outer constraint, the solution may be distorted. For example, if the outer mechanism imposes a minimum reserve price above the Myerson optimal reserve, the inner mechanism must use the suboptimal reserve, and both expected revenue and allocative efficiency may suffer.

Regulatory design: the outer layer's problem

The outer designer (regulator) solves:

$$\max_{\mathcal{M}' \subseteq \mathcal{M}} W(\mathcal{M}', M^{*\text{in}}(\mathcal{M}'), \sigma^*(M^{*\text{in}}))$$

where W is the regulator's welfare function (social welfare, consumer surplus, revenue, etc.) and $M^{*\text{in}}(\mathcal{M}')$ is the inner designer's best response to the constraint set $\mathcal{M}'$.

Theorem (Layered Revenue-Welfare Tradeoff). Let W be a weighted social welfare function and V^{in} be the inner designer's revenue. For any outer constraint set $\mathcal{M}'$, if the inner designer maximizes revenue subject to $\mathcal{M}'$, and the outer designer maximizes welfare, then the equilibrium outcome satisfies:

$$W(\mathcal{M}'^*, M^{*\text{in}}) \leq W^{\text{first-best}}$$

with equality only if the inner designer's revenue-maximizing mechanism within $\mathcal{M}'^*$ happens to coincide with the welfare-maximizing mechanism. Generically, there is a strict welfare loss from the layered structure — the "cost of delegation."

Information flow between layers

Define the **information partition** at each layer: - Agents observe their own types θ_i and the mechanism rules M^{in}. - The inner designer observes the outer constraint $\mathcal{M}'$ and (possibly) aggregate statistics of agent behavior. - The outer designer observes the inner mechanism choice M^{in} and (possibly) the allocation outcome.

The flow is:

$$\theta_i \xrightarrow{\text{agents}} \sigma_i^{\text{in}} \xrightarrow{\text{inner mechanism}} \text{outcome} \xrightarrow{\text{reports}} \text{outer designer}$$

Each arrow is a strategic communication channel: agents may lie to the inner mechanism (standard IC problem), and the inner designer may distort reports to the outer designer (regulatory evasion). The layered mechanism must be incentive-compatible at **both** levels simultaneously, which is a strictly harder requirement than IC at either level alone.

The tournament-within-casino layered structure

Formalize a poker tournament as a layered mechanism. Let the casino be the outer mechanism with rules $R^{\text{out}} = (\text{rake } r, \text{max entries } \bar{n}, \text{guarantee } G)$. The tournament director is the inner mechanism choosing blind structure $B = (b_1, b_2, \ldots, b_T)$, payout structure $P = (p_1, \ldots, p_k)$, and starting stack c_0, subject to R^{out}.

The inner designer's objective: maximize expected entries (which determines the inner mechanism's revenue $n \cdot \text{buy-in}$) subject to the casino's rake and guarantee constraints. The agents (players) choose whether to enter and how to play, given the full specification (B, P, c_0).

The ICM (Independent Chip Model) introduces a nonlinear payoff transformation $u_i^{\text{ICM}}(c_i, \mathbf{c}_{-i}, P)$ where c_i is player i's chip count and P is the payout structure. This transformation is a feature of the inner mechanism (the tournament structure), not of the raw poker game. It creates strategic incentives — tighter play near the bubble, risk aversion even for risk-neutral players — that exist only because of the layered mechanism.

Worked Example

Spectrum auction under regulatory caps

Outer mechanism (regulator). The FCC imposes: (i) a spectrum cap of 100 MHz per bidder, (ii) a minimum reserve price of $10M per license, (iii) a prohibition on bidder collusion, (iv) a requirement of simultaneous ascending auction format.

Inner mechanism (auctioneer). Subject to these constraints, the auctioneer chooses: activity rules, bid increments, package-bidding provisions, and information-revelation policy (which bids are publicly visible during the auction).

Agents (telecom firms). Each firm has a private valuation θ_i for each license and bids strategically in the ascending auction.

Without the outer mechanism. The auctioneer would run an unconstrained Myerson-optimal auction: set individual reserve prices at $r_i^* = \theta_0 + (1 - F(\theta_0))/f(\theta_0)$ (virtual-value formula from 6.3), use a sealed-bid format for revenue maximization, and impose no spectrum caps.

With the outer mechanism. The spectrum cap constrains the allocation space (no firm can win too much). The minimum reserve may be above or below the Myerson optimal reserve. The format constraint (ascending auction) sacrifices some revenue relative to optimal sealed-bid but has transparency advantages the regulator values. The anti-collusion constraint affects information policy: showing too many bids may facilitate tacit collusion even without explicit communication.

Layered equilibrium. The auctioneer, constrained to ascending format with caps, chooses tight activity rules and limited information revelation to maximize revenue. The firms bid, adjusting for the caps (which reduce demand for very large firms and create opportunities for smaller firms). The regulator, anticipating this, chose the caps to balance efficiency (large firms may be more efficient) against competition (market concentration is socially costly). The outcome is neither the revenue-maximizing outcome nor the welfare-maximizing outcome, but the equilibrium of the layered system.

Welfare loss from layering. Suppose the Myerson optimal auction without caps generates expected revenue $500M and welfare $800M. The constrained auction generates revenue $450M and welfare $750M. The $50M revenue loss is the "cost of regulation" borne by the auctioneer; the $50M welfare loss is the cost of the caps. But the regulator values competition, which the caps promote, so the regulator's augmented welfare may be higher. The tradeoff is intrinsic to the layered structure.

Poker Anchor

Concept mapping

Formal object	Poker instantiation
Outer mechanism M^{out}	Casino rules: rake, game-spread rules, licensing, tournament overlay/guarantee
Inner mechanism M^{in}	Tournament structure: blind schedule, payout table, starting stacks, clock, late registration
Outer designer	Casino management, gaming commission
Inner designer	Tournament director

Formal object	Poker instantiation
Agent-level game	The poker hands played within the tournament
Layered constraint	Casino rake constrains tournament prize pool; blind structure constrains hand-level strategy
ICM transformation	Payout structure creates nonlinear chip-to-dollar mapping — a meta-game over the hand game

Concrete situation: blind structure as inner mechanism design

A tournament director designs a 12-level blind structure for a $200 buy-in daily tournament. The casino's outer constraints: the tournament must finish within 5 hours (table rental cost), the rake is 15% ($30 of the $200 goes to the house), and a $10,000 guarantee must be met. Given these constraints, the director chooses starting stack (15,000 chips), blind levels (20 minutes each), and payout structure (top 15% of field).

The blind structure is a mechanism that determines the pace of elimination and the relative importance of skill versus variance. Fast blinds (turbo) increase variance, attract recreational players (who enjoy the "lottery" feel), and finish within the time constraint. Slow blinds increase the skill component, attract regulars, but risk running over the time limit.

The inner designer's tradeoff: fast blinds fill the guarantee (more recreational entries) but reduce the skill edge that attracts regulars; slow blinds attract regulars but may not fill the guarantee if the recreational pool is thin. This is a constrained mechanism-design problem — the inner designer optimizes within the outer constraints.

The ICM layer on top of the payout structure creates a second mechanism layer. Near the money bubble, a player's optimal strategy depends not on chip EV (the hand-level game) but on dollar EV (the tournament-level game), which is a nonlinear function of all stack sizes and the payout table. This is mechanism design within mechanism design: the payout structure transforms hand-level poker into a different strategic game with different equilibria.

What this understanding buys you

Tournament selection is mechanism analysis. When choosing which tournament to enter, a skilled player should analyze the layered mechanism: how does the blind structure interact with the payout table? How does the rake constrain the overlay? How does the starting stack interact with the blind schedule to determine how many "playable" levels exist before the game becomes a push-fold exercise? These are not fuzzy qualitative questions — they are quantifiable properties of the layered mechanism.

ICM awareness is layer awareness. Most intermediate players learn ICM as a "tool" — plug in stack sizes, get a payout equity. The deeper insight is that ICM is a **layer** — it is the transformation that the payout mechanism imposes on the hand game. Understanding this lets you see when ICM matters (near pay jumps, short stacks, final table) and when it does not (deep in the tournament, far from the money). It also lets you see that ICM is not a law of nature but a consequence of a specific payout structure — change the payout structure (winner-take-all vs flat) and ICM changes correspondingly.

Regulatory arbitrage between layers. Different casinos impose different outer constraints, leading to different inner mechanisms. A player who understands the layered structure can identify which casino's tournament program produces the most favorable inner mechanism for their skill profile. This is 10.2 applied to bankroll optimization.

Cross-Links

- **6.1–6.3 Mechanism design** — the single-layer theory that 10.2 stacks.

- **10.1 Games embedded in institutions** — the general nesting framework; 10.2 specializes to mechanisms within mechanisms.
- **10.3 Contracts and principal-agent** — the bilateral special case of layered mechanisms.
- **4.3 Bayesian Nash equilibrium** — the solution concept at the agent level of each layer.
- **10.6 Strategic ecosystems** — the many-layer generalization.
- **Economics / regulation theory** — Laffont-Tirole regulatory economics as layered mechanism design.

Structural Compression

Invariant: Real mechanisms are never designed in isolation; every mechanism sits inside an outer mechanism that constrains the designer's choices, creating a layered optimization problem where each layer's equilibrium depends on the others, and the overall outcome is the fixed point of the nested strategic interaction.

Mechanism: The outer mechanism constrains the set of available inner mechanisms; the inner designer optimizes within this constraint set; agents play the resulting game; information and incentives propagate bidirectionally between layers; the layered equilibrium is the simultaneous solution of all layers' optimization problems.

Cross-System Echo: Layered mechanisms are structurally identical to **compiler optimization within hardware constraints** in computer science (the compiler optimizes code subject to instruction-set constraints imposed by the chip architecture, which is itself designed anticipating compiler behavior) and to **gene regulation within cellular constraints** in molecular biology (transcription factors optimize gene expression within the regulatory architecture of the cell, which is itself the product of evolutionary selection on the regulatory architecture). In each case, the optimization at the inner level is meaningful only given the constraints at the outer level, and the constraints at the outer level are meaningful only given the response at the inner level.

Exercises

1. Formalize a layered mechanism where the outer mechanism is a government tax on auction revenue and the inner mechanism is a first-price sealed-bid auction. Derive the optimal reserve price as a function of the tax rate and show that it differs from the Myerson optimal reserve.

2. A casino sets rake r and a tournament director designs payout structure P. Show that for a fixed buy-in B, the prize pool is $(1 - r) \cdot B \cdot n$ where n is the number of entrants, and that the payout structure P is constrained by this total. Derive the ICM equity of a player with fraction f of total chips in a winner-take-all versus a flat-payout structure.

3. Consider a two-layer mechanism where the outer layer imposes a maximum price (price cap) and the inner layer is a monopolist choosing quantity. Show that the layered equilibrium differs from both the unconstrained monopoly outcome and the competitive outcome. Characterize the welfare as a function of the price cap.

4. Prove that adding an outer-layer constraint to a revenue-maximizing inner mechanism can never increase inner revenue (the inner designer's problem is weakly harder with more constraints). State conditions under which it strictly decreases revenue.

5. In a poker tournament with payout structure $P = (50\%, 30\%, 20\%)$ for three players, compute the ICM equity for each player as a function of chip stacks (c_1, c_2, c_3) with $c_1 + c_2 + c_3 = T$. Show that ICM equity is concave in own chip count and derive the implication for risk-taking at the final table.

6. Two mechanisms are layered: the outer mechanism is a Vickrey auction selecting which of two firms gets the right to run a monopoly, and the inner mechanism is the winning firm's pricing decision. Compare the layered equilibrium to a direct mechanism where the regulator runs both the auction and the pricing. Under what conditions does the direct mechanism dominate?

7. **Argue, structurally,** why ICM is not a feature of poker but a feature of the tournament mechanism layered on top of poker. Identify what changes about ICM if the payout structure changes, and explain why this shows that ICM is a property of the layer, not the game.

Game Theory · Module 10 — Strategic Systems Integration · Node 10.2
Concepts: mechanism-design, incentive-compatibility, hierarchical-systems, principal-agent
Prerequisites: 6.1, 6.2, 6.3, 10.1
Enables: 10.3, 10.6

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