Hawk-Dove and Asymmetric Contests

Game Theory · Module 7 — Evolutionary Game Theory

Node 7.3

Position in Knowledge Graph

System: Game Theory **Structural Domain:** Evolutionary Game Theory **Node:** 7.3

The Hawk-Dove game is the canonical model of evolutionary game theory — the Prisoner's Dilemma of biology. Every feature of the theory developed in 7.1 (replicator dynamics) and 7.2 (ESS) finds its cleanest illustration here: an interior mixed ESS, an explicit replicator trajectory, and a precise prediction of population composition from the payoff parameters. But the deeper lesson of 7.3 comes from its second half: **asymmetric contests**. When players differ in a payoff-irrelevant way — one is the "owner" and the other the "intruder," or one arrived first — a new class of strategies becomes available: **conditional strategies** that condition on the asymmetry. The most important is the **bourgeois strategy** ("if owner, play Hawk; if intruder, play Dove"), which Maynard Smith showed can be an ESS even when ownership confers no intrinsic advantage. The asymmetric contest framework explains territorial behavior in biology, incumbency advantages in economics, and the profound role of position in poker. It also provides the first worked example of a game where the symmetric ESS and the asymmetric ESS coexist and differ.

Formal Definition

Symmetric Hawk-Dove

Two players contest a resource of value $V > 0$. Each chooses **Hawk** (escalate: fight for the resource) or **Dove** (display: share or retreat). Payoff matrix:

$$A = \begin{pmatrix} (V-C)/2 & V \\ 0 & V/2 \end{pmatrix}$$

where $C > 0$ is the cost of fighting (incurred only when two Hawks meet, split symmetrically). Rows and columns: strategy 1 = Hawk, strategy 2 = Dove.

Payoff logic. Hawk-Hawk: fight, each wins half the time (gets V) and loses half the time (pays C), expected $(V-C)/2$. Hawk-Dove: Hawk takes the resource, Dove retreats. Dove-Dove: display, share the resource equally.

Asymmetric Hawk-Dove (the owner-intruder game)

Now suppose each contest has a **role asymmetry**: one player is the "owner" (arrived first, or is larger, or holds a territory) and the other is the "intruder." With probability 1/2, a given individual is the owner; with probability 1/2, the intruder. The asymmetry may or may not affect payoffs.

A **conditional strategy** specifies an action for each role. There are four:

- **Hawk-Hawk (HH):** play Hawk regardless of role.
- **Dove-Dove (DD):** play Dove regardless of role.
- **Bourgeois (B):** play Hawk if owner, Dove if intruder.

- **Anti-bourgeois (AB):** play Dove if owner, Hawk if intruder.

The payoff matrix for the asymmetric game (expected payoff, averaged over the two equally likely role assignments) is a 4×4 matrix derived from the underlying 2×2 Hawk-Dove payoffs.

Definition (Bourgeois strategy). The bourgeois strategy B is: "if I am the owner, escalate; if I am the intruder, defer." In the asymmetric Hawk-Dove game, it is an ESS under the condition $V < C$, even when ownership confers no fitness advantage beyond the strategic convention.

Definition (Correlated asymmetry, Maynard Smith 1982). A **correlated asymmetry** is any observable difference between the contestants (owner vs intruder, large vs small, first-mover vs second-mover) that is common knowledge and payoff-irrelevant (or at least not fully determined by payoffs). Strategies can condition on the asymmetry, expanding the strategy space and potentially creating new ESS.

Intuition

The symmetric Hawk-Dove game captures the fundamental tradeoff in animal conflict: escalating gives you the resource if the opponent backs down, but costs you dearly if the opponent also escalates. When fighting is cheap ($V > C$), the cost is worth it and everyone escalates — all-Hawk is ESS. When fighting is expensive ($V < C$), the cost exceeds the prize, and pure Hawk cannot be ESS (a pair of Hawks both lose on average). Nor can pure Dove be ESS (a single Hawk invades a population of Doves and takes everything). The ESS is a mixture: a fraction V/C of the population plays Hawk, the rest play Dove. The population reaches a balance where the expected payoff to Hawk equals the expected payoff to Dove.

The asymmetric version adds a qualitatively new phenomenon. Suppose fighting is expensive ($V < C$), but before each contest one player is designated "owner" and the other "intruder." Now the bourgeois strategy — "fight if I own, yield if I do not" — is an ESS. In a population of bourgeois players, contests are settled without fighting: the owner keeps the resource, the intruder defers. The average payoff is $V/2$ (each individual is owner half the time), which is the same as Dove-Dove, but the bourgeois convention is *stable* against invasion in a way that Dove-Dove is not.

The biological significance: territorial behavior in animals is explained not by any intrinsic advantage of ownership, but by the stability of the bourgeois convention. A spider that holds a web defends it aggressively; one that encounters an occupied web retreats. This is not because the web gives the holder a fighting advantage, but because the convention "owner fights, intruder yields" is an ESS. The convention creates the territorial behavior, not the other way around.

The anti-bourgeois strategy — "yield if owner, fight if intruder" — is the mirror image. It is also an ESS under $V < C$, but it is far rarer in nature. The reason is that the owner typically has better information about the resource (having already invested in it), making the bourgeois convention a Schelling focal point. But the mathematical structure admits both.

Structural Role

7.3 is the canonical worked example for Module 7, analogous to the Prisoner's Dilemma in Module 5:

- **7.1 Replicator dynamics** — 7.3 provides the first game where the replicator is solved explicitly and the interior trajectory is computed.
- **7.2 ESS** — 7.3 provides the first full verification of ESS conditions for both a mixed strategy (symmetric case) and a conditional strategy (bourgeois).
- **7.4 Population games** — 7.3's asymmetric extension motivates the multi-population framework.
- **7.6 Adaptive dynamics** — the V/C ratio is the first example of an "invasion fitness gradient" governing the direction of trait evolution.

- **Module 3 Extensive-form games** — the asymmetric Hawk-Dove is naturally modeled as a sequential game with an information structure (who is the owner?), connecting evolutionary and extensive-form perspectives.

The asymmetric version introduces a structural idea that pervades the rest of the course: **role-contingent strategies** can be ESS even when the role carries no intrinsic advantage, because the role serves as a coordination device. This is the evolutionary foundation of institutions, norms, and positional authority.

Mathematical Development

Symmetric Hawk-Dove: replicator analysis

Let $x = x_H$ be the frequency of Hawk. Then $1 - x$ is the frequency of Dove. The fitnesses are:

$$f_H(x) = \frac{V-C}{2}x + V(1-x) = V - \frac{V+C}{2}x,$$
$$f_D(x) = 0 \cdot x + \frac{V}{2}(1-x) = \frac{V}{2}(1-x).$$

Mean fitness: $\bar{f} = x f_H + (1-x) f_D$. The replicator equation in the single variable x:

$$\dot{x} = x(1-x)(f_H - f_D).$$

Compute $f_H - f_D = V - \frac{V+C}{2}x - \frac{V}{2}(1-x) = \frac{V}{2} - \frac{C}{2}x = \frac{1}{2}(V - Cx)$.

So:

$$\dot{x} = \frac{x(1-x)}{2}(V - Cx).$$

Rest points: $x = 0$ (all Dove), $x = 1$ (all Hawk), and $x^* = V/C$ (interior, exists iff $V < C$).

Stability (case $V < C$): - At $x = 0$: $\dot{x} \approx (V/2)x > 0$, so all-Dove is unstable (Hawk invades). - At $x = 1$: $\dot{x} \approx -(C-V)/2(1-x) < 0$ for x slightly below 1, so all-Hawk is unstable (Dove invades). - At $x^* = V/C$: linearize. $d/dx[\dot{x}]$ at x^* gives a negative eigenvalue (the coefficient of the perturbation is $-Cx^*(1-x^*)/2 < 0$). So the interior is **asymptotically stable**. Confirmed: $x^* = V/C$ is the ESS.

Stability (case $V > C$): The interior point $V/C > 1$ is outside the simplex. All-Hawk $x = 1$ is the unique stable equilibrium. $\dot{x} > 0$ for all $x \in (0, 1)$, so Hawk takes over.

Asymmetric Hawk-Dove: the four-strategy game

In the asymmetric version, the four strategies are HH, DD, B, AB. The expected payoff of strategy i against strategy j is the average over the two role assignments (each player is owner with probability 1/2):

$$U(i,j) = \frac{1}{2}u(\alpha_i^O, \alpha_j^I) + \frac{1}{2}u(\alpha_i^I, \alpha_j^O)$$

where α_i^O is the action of strategy i in the owner role and α_i^I in the intruder role.

Computing the 4×4 payoff matrix (using symmetric Hawk-Dove payoffs):

$$U = \begin{pmatrix} (V-C)/2 & V & (V+(V-C)/2)/2 & ((V-C)/2+V)/2 \\ 0 & V/2 & V/4 & V/4 \\ ((V-C)/2+0)/2 & (V+V/2)/2 & V/2 & (V-C)/4 \\ (0+V)/2 & (0+V/2)/2 & (V-C)/4 & V/2 \end{pmatrix}.$$

Simplifying (case $V < C$):

$$U(B,B) = \frac{1}{2}(V+0) = \frac{V}{2}, \qquad U(HH,B) = \frac{1}{2}\left(\frac{V-C}{2} + V\right) = \frac{3V-C}{4}.$$

Since $V < C$, we have $(3V-C)/4 < V/2$ iff $3V - C < 2V$ iff $V < C$. True. So HH cannot invade B.

$$U(DD,B) = \frac{1}{2}(0 + V/2) = \frac{V}{4} < \frac{V}{2} = U(B,B).$$

DD cannot invade B either.

$$U(AB,B) = \frac{1}{2}\left(0 + \frac{V-C}{2}\right) = \frac{V-C}{4} < 0 < \frac{V}{2} = U(B,B).$$

AB cannot invade B. So B is a strict Nash in the four-strategy game, hence an ESS. The bourgeois strategy, when $V < C$, is an ESS that settles all contests without fighting.

Ownership as coordination device

Theorem (Maynard Smith, 1982). In the asymmetric Hawk-Dove game with $V < C$ and payoff-irrelevant role asymmetry, the bourgeois strategy is an ESS. The anti-bourgeois strategy is also an ESS. The symmetric mixed ESS $x^* = V/C$ from the uncorrelated game is *not* an ESS of the asymmetric game.

The key insight: the role asymmetry creates a focal point for conditional strategies that coordinate behavior more efficiently than any unconditional mixture. The bourgeois convention eliminates all fighting (average payoff $V/2$), while the symmetric mixed ESS has fighting (average payoff $V/2 - V^2/(2C) < V/2$). The bourgeois ESS Pareto-dominates the mixed ESS.

Worked Example

Numerical Hawk-Dove: $V = 4, C = 10$

Symmetric game. Payoff matrix:

$$A = \begin{pmatrix} -3 & 4 \\ 0 & 2 \end{pmatrix}.$$

ESS: $x^* = V/C = 0.4$. In a population of 40% Hawks and 60% Doves, both strategies earn the same fitness.

Verify: $f_H(0.4) = -3(0.4) + 4(0.6) = -1.2 + 2.4 = 1.2$. $f_D(0.4) = 0(0.4) + 2(0.6) = 1.2$. Equal.

Mean fitness: $\bar{f} = 0.4(1.2) + 0.6(1.2) = 1.2$.

Replicator trajectory. Starting from $x_0 = 0.8$ (too many Hawks):

$$\dot{x} = \frac{0.8 \cdot 0.2}{2}(4 - 10 \cdot 0.8) = 0.08 \cdot (-4) = -0.32.$$

Hawk frequency is decreasing — too many Hawks means Hawks are below average fitness. The system approaches $x^* = 0.4$ from above.

Starting from $x_0 = 0.1$ (too few Hawks):

$$\dot{x} = \frac{0.1 \cdot 0.9}{2}(4 - 1) = 0.045 \cdot 3 = 0.135.$$

Hawk frequency increasing. The system approaches $x^* = 0.4$ from below.

Asymmetric game. The bourgeois strategy gives payoff $V/2 = 2$ against itself (no fighting). Compare to the symmetric ESS mean fitness 1.2. The bourgeois convention is better for everyone — it eliminates the wasteful Hawk-Hawk fights that cost $C = 10$ half the time.

Bourgeois ESS check. Against bourgeois, the best invader is HH with payoff $(3 \cdot 4 - 10)/4 = 0.5 < 2$. All other strategies score less than 2. Bourgeois is a strict Nash, hence ESS.

Poker Anchor

Concept mapping

Formal object	Poker instantiation
Hawk	Aggressive play: 3-bet, c-bet, barrel, overbet
Dove	Passive play: call, check, fold to aggression
Cost C	Risk of large loss when both players escalate (cooler, big bluff into big bluff)
Value V	Pot size / expected gain from winning the contest
Mixed ESS V/C	Optimal aggression frequency as a function of pot odds and variance tolerance
Owner role	Player with positional advantage (IP, or in a re-raised pot, the 3-bettor)
Intruder role	Player without positional advantage (OOP, or the cold-caller)
Bourgeois strategy	"When IP, play aggressively; when OOP, play passively"

Concrete situation: positional Hawk-Dove

In a heads-up pot, the in-position (IP) player has a structural advantage: they act last, see more information, control the pace. The out-of-position (OOP) player is at a disadvantage. A standard result from solver outputs: the IP player c-bets more frequently, uses larger sizings, and applies more pressure. The OOP player checks more, calls more, and rarely leads.

This is the bourgeois strategy in the Hawk-Dove framework. Position is the "ownership asymmetry." It is not payoff-irrelevant (IP genuinely has an informational edge), but the strategic convention amplifies the asymmetry far beyond its intrinsic value. Solver solutions often have the IP player betting at high frequency even with weak hands (Hawk behavior), while the OOP player checks even with medium-strong hands (Dove behavior). The convention — "the IP player is the aggressor" — is an ESS because deviating from it (e.g., OOP leading into the IP player) exposes you to exploitation.

Concrete situation: the "who blinks first" dynamic

Two aggressive regulars in a high-stakes cash game. Both want to play Hawk (aggressive), but when both escalate (mutual 4-bet bluffs, 5-bet shoves), the variance cost C is enormous. The symmetric ESS says: each should escalate with probability V/C, mixing aggression and passivity. In practice, players develop a bourgeois-like convention: "whoever was the initial aggressor (the raiser) gets to be Hawk in the later streets; the caller defers." This is positional bourgeois applied dynamically across streets. It reduces variance while preserving the ability to fight when holding a strong hand.

What this understanding buys you

Aggression is not free. The Hawk-Dove framework quantifies the tradeoff: aggression wins against passivity, but mutual aggression is destructive. Optimal aggression frequency is V/C, which decreases as variance (C) increases. High-stakes players who reduce aggression in high-variance spots are not "nitting up" — they are playing the ESS.

Position is the ownership asymmetry. The single strongest predictor of who "should" be aggressive in a poker hand is position. The bourgeois strategy — aggressive IP, passive OOP — is evolutionarily stable. Players who deviate (aggressive OOP without a strong hand, passive IP without a weak hand) are playing anti-bourgeois, which is exploitable in most structures.

Territorial conventions are self-enforcing. Once the bourgeois convention is established (at a table, in a player pool, in a metagame era), it is stable. New players who try to "fight for territory" (play aggressively from bad position) lose to the convention. The convention persists not because anyone chose it, but because it is an ESS.

Cross-Links

- **7.1 Replicator dynamics** — the explicit trajectory for Hawk-Dove is a solved replicator equation.
- **7.2 ESS** — Hawk-Dove is the standard example for ESS computation.
- **7.4 Population games** — the asymmetric version motivates multi-population dynamics.
- **3.2 Information sets** — the asymmetric game has an information structure (each player observes their role).
- **4.1 Bayesian games** — the role assignment is formally a type, and bourgeois is a Bayesian Nash strategy.
- **5.3 Trigger strategies** — bourgeois resembles a trigger strategy: "if I am the incumbent, defend."
- **Module 10 Strategic systems** — conventions and institutions as ESS of asymmetric games.

Structural Compression

Invariant: The symmetric Hawk-Dove game with $V < C$ has a unique interior ESS $x^* = V/C$; the asymmetric extension with role labels creates the bourgeois strategy (Hawk-if-owner, Dove-if-intruder), which is an ESS that eliminates all fighting and Pareto-dominates the symmetric mixed ESS; the role asymmetry need not be payoff-relevant — it serves purely as a coordination device.

Mechanism: In the symmetric game, the replicator $\dot{x} = x(1-x)(V - Cx)/2$ drives the population to the interior ESS. In the asymmetric game, the expanded strategy space (conditional on role) admits strict Nash equilibria (bourgeois, anti-bourgeois) that reduce the cost of conflict to zero by assigning aggression and deference based on observable role.

Cross-System Echo: Bourgeois strategies are the evolutionary analogue of **property rights** in economics and **conventions** in philosophy (Lewis, 1969). A property right is a socially enforced bourgeois strategy: "the owner has the right to exclude, the non-owner must defer." The right need not be efficient in any deep sense — it is simply the ESS of the underlying asymmetric contest. The same structure appears in **TCP congestion control**: the "owner" (established flow) maintains its rate; the "intruder" (new flow) ramps up slowly. And in **territorial behavior** in biology: the resident fights, the newcomer retreats.

Exercises

1. For the symmetric Hawk-Dove game with general $V, C > 0$, compute the interior ESS x^*, the mean population fitness at x^*, and the mean fitness under the bourgeois convention. Show the bourgeois

convention always Pareto-dominates the symmetric ESS.

2. Solve the replicator equation $\dot{x} = x(1-x)(V - Cx)/2$ by separation of variables. Express $x(t)$ in closed form as a function of x_0, V, C, t.
3. Verify that the anti-bourgeois strategy is also an ESS of the asymmetric Hawk-Dove game with $V < C$. Why is it observed less frequently in nature than bourgeois?
4. Construct the full 4×4 payoff matrix for the asymmetric Hawk-Dove game with $V = 4, C = 10$. Confirm that bourgeois is a strict Nash equilibrium and compute the invasion barrier for each alternative strategy.
5. Modify the Hawk-Dove game so that owners have a slight fighting advantage: if owner is Hawk and intruder is Hawk, the owner wins with probability $p > 1/2$. How does this affect the ESS set? Show that for p close to 1, the bourgeois strategy remains ESS but anti-bourgeois may no longer be ESS.
6. Consider a three-strategy variant: Hawk, Dove, and Retaliator (plays Dove initially, switches to Hawk if the opponent plays Hawk). Show that Retaliator weakly dominates Dove and analyze the ESS set.
7. **Argue, structurally,** why the bourgeois strategy is an ESS even when ownership confers no intrinsic advantage. What is the minimal ingredient needed for the bourgeois convention to stabilize? Connect to the concept of correlated equilibrium (Module 4) and to the role of observable asymmetries as coordination devices in real-world institutions.

Game Theory · Module 7 — Evolutionary Game Theory · Node 7.3
Concepts: evolutionary-stability, nash-equilibrium, replicator-dynamics
Prerequisites: 7.1, 7.2
Enables: 7.4, 7.5, 7.6

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