Lesson 2.1 — Algebra Sentences: Reading Equations as Claims About Relationships

Mathematical Intuition · Module 2 — Equation Literacy

Node 2.1

1. Why this matters (Hook)

You can already add, multiply, divide, exponentiate, take logs, and differentiate.

But open any physics textbook or ML paper and you hit a wall of symbols. The wall is not the math. The wall is that nobody taught you to **read**.

An algebraic equation is a **sentence**. It has a subject (the quantity being described), a verb (the operation), and an object (the other quantities it relates to).

$$y = mx + b$$

This is not instructions to "solve for y." This is a **claim**: "y is determined by a linear scaling of x, shifted by a baseline b."

Once you learn to read algebra as claims about relationships, every equation in every field becomes a sentence you can parse before you compute anything.

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2. Core Intuition

2.1 Every equation is a claim

The equals sign does not mean "calculate." It means **"these two things are the same."**

$$F = ma$$

Claim: "The force experienced by an object is its mass scaling its acceleration."

$$E = mc^2$$

Claim: "Energy is mass, scaled by the square of the speed of light."

Before you ever plug in numbers, you should be able to state the claim in plain language.

2.2 Variables are roles, not mysteries

Students often treat variables as unknowns to be solved. But variables are **roles in a story**.

In $PV = nRT$: - P plays the role of pressure - V plays the role of volume - T plays the role of temperature - n and R set the scale

The equation says: "Pressure and volume, acting together, are proportional to temperature."

When you see a variable, ask: **what role does this play?**

2.3 Coefficients are importance weights

A coefficient tells you **how strongly** one quantity influences another.

In $y = 3x + 7$: - The 3 says: "x's influence on y is tripled." - The 7 says: "even when x contributes nothing, y starts at 7."

In $F = -kx$ (Hooke's law): - The k says: "the spring's stiffness determines how strongly displacement creates force." - The minus sign says: "the force opposes the displacement."

Signs, magnitudes, and units of coefficients carry the physics.

2.4 Equations have grammar

Every algebraic sentence follows a grammar:

Grammar element	Algebraic form	Meaning
Subject	Left-hand side	The quantity being described
Verb	$=$, $\leq$, $\propto$	The relationship type
Object	Right-hand side	What determines the subject
Adjective	Coefficient	How strongly
Conjunction	$+$	"and also"
Negation	$-$	"minus the effect of"
Grouping	Parentheses	"treat this sub-expression as one unit"

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3. Formal Lens (Light)

3.1 Linear equations: proportional claims

$$y = mx + b$$

Sentence: "y is proportional to x (scaled by m), shifted by a baseline b."

The slope m is the **rate** — how much y changes per unit change in x. The intercept b is the **starting point** — y's value when x contributes nothing.

Every linear model, from Ohm's law $V = IR$ to consumption functions $C = C_0 + cY$, is an instance of this one sentence.

3.2 Quadratics: curvature claims

$$y = ax^2 + bx + c$$

Sentence: "y has a curved relationship with x: a controls the curvature, b controls the tilt, c sets the baseline."

- $a > 0$: the curve opens upward (has a minimum)
- $a < 0$: the curve opens downward (has a maximum)
- The vertex is where the competing influences of ax^2 and bx balance

This is the simplest equation that can describe **optimization** — a quantity that has a best value.

3.3 Systems: simultaneous claims

$$\begin{cases} 2x + 3y = 7 \\ x - y = 1 \end{cases}$$

Sentence: "Two constraints must hold at the same time. The solution is the point where both claims are true simultaneously."

Geometrically: each equation is a line; the solution is their intersection. This generalizes: n equations in n unknowns = n surfaces intersecting at a point.

3.4 Inequalities: constraint claims

$$x^2 + y^2 \leq r^2$$

Sentence: "The point (x, y) must lie inside or on the circle of radius r."

Inequalities define **regions**, not points. In optimization, constraints carve out the feasible set — the territory where solutions are allowed to live.

3.5 Functional equations: structural claims

$$f(x + y) = f(x) \cdot f(y)$$

Sentence: "The function converts addition into multiplication."

This is not asking you to solve for f. It is a **structural claim** about what kind of function f must be. (The answer: exponentials. Only $f(x) = a^x$ satisfies this.)

3.6 Polynomial factoring: decomposition claims

$$x^2 - 1 = (x - 1)(x + 1)$$

Sentence: "The expression $x^2 - 1$ is zero exactly when x = 1 or x = -1."

Factoring is not a calculation trick. It is a **decomposition** — breaking a complex claim into simpler sub-claims.

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4. Cross-Domain Examples

4.1 Physics — the ideal gas law

$$PV = nRT$$

Reading: "Pressure times volume (the mechanical state) equals the number of particles times a constant times temperature (the thermal state). Mechanical and thermal descriptions of the same gas must agree."

The equals sign here is a bridge between two ways of describing reality.

4.2 Economics — supply and demand equilibrium

$$Q_s(p) = Q_d(p)$$

Reading: "The quantity suppliers want to sell at price p equals the quantity buyers want to buy. The equilibrium price is where these two stories agree."

The variable p is not being "solved for" — it is the price at which two independent claims become simultaneously true.

4.3 Optimization — the normal equations

$$X^T X\hat{\beta} = X^T y$$

Reading: "The best linear fit $\hat{\beta}$ is the one where the data matrix's own structure (via $X^T X$) times the coefficients equals the data matrix's correlation with the target (via $X^T y$)."

This is a claim about balance: the model's internal geometry must match the data's signal.

4.4 Quantum Mechanics — eigenvalue equation

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

Reading: "The Hamiltonian acting on the state $|\psi\rangle$ produces the same state back, scaled by E. The state is a fixed point of the energy operator, and E is the energy."

The equals sign is a **consistency condition**: the state must be compatible with the operator.

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5. Micro Drills

Answer in words, not calculations.

1. **Read aloud:**
$$v = v_0 + at$$
State the claim this equation makes about velocity, in one sentence.

2. **Identify the grammar:** In $F = -kx$, what is the subject? The verb? The object? What does the minus sign mean?

3. **Spot the constraint:**
$$x_1 + x_2 + x_3 = 1, \quad x_i \geq 0$$
What kind of object does this define? (Hint: it lives in 3D space.)

4. **Translate to algebra:** "The population next year equals this year's population plus births minus deaths." Write the equation and identify each term's role.

5. **Structural claim:**
$$f(xy) = f(x) + f(y)$$
What kind of function must f be? Why does this say "multiplication becomes addition"?

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6. Reflection

Pick one equation from your current work — physics, ML, quantum computing, anything.

Without computing, write three sentences: - What claim does the equation make? - What role does each variable play? - What would it mean if the equation were violated?

If you can answer these, you can read algebra.

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Prerequisites: 1.1, 1.2, 1.3, 1.4, 1.5, 1.6
Enables: 2.2, 2.3, 2.4, 2.5, 2.6

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