Lesson 5.2: Translate Stories

Mathematical Intuition · Module 5 — Clarity Drills

Node 5.2

Overview

Practice converting word problems and physical scenarios into mathematical expressions and vice versa.

Learning Objectives

- Translate verbal descriptions into equations
- Convert equations into plain language explanations
- Recognize mathematical structure in real-world scenarios
- Build fluency in moving between representations

Drill Format

Story → Equation

Given a verbal description, write the mathematical expression.

Equation → Story

Given an equation, explain what it describes in plain language.

Drill Set 1: Story → Equation

Problem 1

Story: "The total cost is $50 plus $20 per hour."

Translation: `C = 50 + 20h` - C: total cost ($) - h: number of hours - 50: fixed cost - 20: cost per hour

Problem 2

Story: "The population doubles every 3 years."

Translation: `P(t) = P₀·2^(t/3)` - P(t): population at time t - P_0: initial population - $2^{(t/3)}$: doubling factor (exponential growth)

Problem 3

Story: "The probability of both events happening is the product of their individual probabilities if they're independent."

Translation: `P(A ∩ B) = P(A)·P(B)` - P(A ∩ B): probability of both A and B - P(A), P(B): individual probabilities - Assumes independence

Problem 4

Story: "The force on a charged particle is proportional to the electric field and the particle's charge."

Translation: `F = qE` - F: force (vector) - q: charge (scalar) - E: electric field (vector)

Problem 5

Story: "The average value of a function over an interval is the integral divided by the interval length."

Translation: $\langle$`f`$\rangle$ `= (1/(b-a))` $\int_a^b$ `f(x)dx` - $\langle$f$\rangle$: average value - [a,b]: interval - Integral divided by length (b-a)

Drill Set 2: Equation → Story

Equation 1: `A = ½bh`

Story: "The area of a triangle is half the product of its base and height." - Geometric formula - Multiplication and division by 2

Equation 2: $\mathtt{v}^2$ `=` $\mathtt{v}_0{}^2$ `+ 2a`Δ`x`

Story: "The final velocity squared equals the initial velocity squared plus twice the acceleration times the displacement." - Kinematic equation - Relates velocity, acceleration, and distance - No time dependence (time eliminated)

Equation 3: `E[X] =` Σ_i $\mathtt{x}_i \cdot$`P(X=`$\mathtt{x}_i$`)`

Story: "The expected value of a random variable is the sum of each possible value weighted by its probability." - Weighted average - Discrete probability distribution

Equation 4: $\nabla\cdot$`E =` ρ/ε_0

Story: "The divergence of the electric field equals the charge density divided by the permittivity of free space." - Gauss's law (differential form) - Relates field to sources

Equation 5: `|`$\psi\rangle$ `=` $\Sigma_n \mathtt{c}_n$`|n`$\rangle$

Story: "A quantum state is a linear combination of basis states, with each basis state weighted by a complex amplitude." - Superposition principle - Expansion in basis

Drill Set 3: Complex Scenarios

Problem 6

Story: "The rate at which a radioactive substance decays is proportional to the amount present."

Translation: `dN/dt = -`λ`N` - dN/dt: rate of change of amount - N: current amount - λ: decay constant (positive) - Negative sign: decreasing

Solution: `N(t) =` $\mathtt{N}_0$`e^(-`λ`t)` (exponential decay)

Problem 7

Story: "The energy of a quantum harmonic oscillator is quantized in units of $\hbar\omega$, with a zero-point energy of $\hbar\omega/2$."

Translation: $\mathtt{E}_n$ `=` $\hbar\omega$`(n + ½)` - n: quantum number (0, 1, 2, ...) - $\hbar\omega$: energy quantum - ½: zero-point contribution

Problem 8

Story: "The gradient descent update moves parameters in the direction opposite to the gradient, scaled by the learning rate."

Translation: θ_new = θ_old - $\alpha \cdot \nabla$L(θ_old) - θ: parameters - α: learning rate - ∇L: gradient of loss - Negative sign: opposite direction

Problem 9

Story: "The probability of measuring a quantum state in a particular eigenstate is the squared magnitude of the overlap between them."

Translation: P(n) = $|\langle n|\psi\rangle|^2$ - P(n): probability of outcome n - $|n\rangle$: eigenstate - $|\psi\rangle$: quantum state - $|\ |^2$: squared magnitude (Born rule)

Problem 10

Story: "The Fourier transform decomposes a time-domain signal into its frequency components by integrating the signal multiplied by complex exponentials."

Translation: F(ω) = $\int_{-\infty}^{\infty}$ f(t)e^(-iωt)dt - F(ω): frequency-domain representation - f(t): time-domain signal - e^(-iωt): complex exponential (basis function)

Drill Set 4: Equation → Story (Advanced)

Equation 6: $[\hat{A},B] = i\hbar$

Story: "The commutator of operators and B equals i$\hbar$, indicating they are canonically conjugate variables with an uncertainty relation." - Non-commuting observables - Quantum uncertainty - Example: position and momentum

Equation 7: $\partial\psi/\partial t = -i\hat{H}\psi/\hbar$

Story: "The time evolution of a quantum state is determined by the Hamiltonian operator acting on the state, with a factor of -i/$\hbar$." - Schrödinger equation - $\hat{H}$: energy operator - Determines dynamics

Equation 8: S = -k_B Σ_i p_i ln(p_i)

Story: "The entropy is the negative sum of each probability multiplied by its natural logarithm, scaled by Boltzmann's constant." - Thermodynamic entropy - Information content - Measures disorder

Equation 9: R_$\mu\nu$ - ½Rg_$\mu\nu$ = (8πG/c^4)T_$\mu\nu$

Story: "The Einstein field equations relate the curvature of spacetime (left side) to the energy-momentum content (right side), with gravitational constant G and speed of light c." - General relativity - Geometry = matter/energy - Tensor equation

Equation 10: $\nabla \times$B = μ_0J + $\mu_0\varepsilon_0\partial$E/$\partial$t

Story: "The curl of the magnetic field equals the current density plus the time rate of change of the electric field, scaled by fundamental constants." - Ampère-Maxwell law - Relates B, J, and changing E - Predicts electromagnetic waves

Practice Exercises

Story → Equation

1. "The kinetic energy is half the mass times the velocity squared."

2. "The period of a pendulum is proportional to the square root of its length divided by gravitational acceleration."
3. "The mutual information between two variables is the difference between their individual entropies and their joint entropy."
4. "The force between two charges is proportional to the product of the charges and inversely proportional to the square of the distance between them."
5. "The variance is the expected value of the squared deviation from the mean."

Equation → Story

6. τ = r×F
7. ΔG = ΔH - TΔS
8. I(X;Y) = H(X) + H(Y) - H(X,Y)
9. $\sigma = \sqrt{(\langle x^2 \rangle - \langle x \rangle^2)}$
10. U(t) = e^(-iĤt/ħ)

Reverse Translation Challenge

For each equation, write a story, then translate back to verify:

1. p = mv
2. W = ∫F·ds
3. $\rho = |\psi\rangle\langle\psi|$
4. L = T - V
5. $\nabla^2 u = 0$

Key Takeaways

- Mathematical expressions encode relationships and processes
- Translation requires identifying quantities, operations, and relationships
- Context matters: same symbol can mean different things in different domains
- Practice both directions: story → equation and equation → story
- Fluency comes from recognizing common patterns

Next Steps

Next, we'll practice spotting generators—operators that generate transformations like time evolution and rotations.

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Concepts: conditional-probability, expectation-value

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