Decoherence Mechanisms

Quantum Foundations · Module 5 — Decoherence

Node 5.4

Position in Knowledge Graph

System: Quantum Foundations **Structural Domain:** Decoherence **Node:** 5.4

Generic system–environment coupling (Node 5.3) produces messy reduced dynamics, but most physical situations are dominated by one of a few canonical channels. The two most important are **dephasing** (pure loss of phase information, no energy exchange) and **amplitude damping** (energy exchange with a zero-temperature environment, leading to relaxation to the ground state). Together they cover the dominant noise processes in almost every quantum hardware platform — superconducting qubits, trapped ions, nitrogen-vacancy centres, photonic qubits — and they are the Lindblad primitives from which more general noise models are built.

Formal Definition

Dephasing channel

The **qubit dephasing channel** with parameter $p \in [0, 1]$ is the CPTP map

$$\Phi_{\text{deph}}(\rho) = (1 - p)\rho + p\, Z\rho Z,$$

with Kraus operators

$$K_0 = \sqrt{1 - p/2}\, I, \qquad K_1 = \sqrt{p/2}\, Z.$$

In matrix form on a qubit,

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \quad \mapsto \quad \Phi_{\text{deph}}(\rho) = \begin{pmatrix} \rho_{00} & (1-p)\rho_{01} \\ (1-p)\rho_{10} & \rho_{11} \end{pmatrix}.$$

Populations (diagonal elements) are preserved. Coherences (off-diagonal elements) are scaled by $1 - p$. In the continuous-time limit, $\rho_{01}(t) = \rho_{01}(0)\, e^{-t/T_2}$, where T_2 is the **dephasing time**.

Amplitude damping channel

The **amplitude damping channel** with parameter $\gamma \in [0, 1]$ is the CPTP map with Kraus operators

$$K_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix}, \qquad K_1 = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix}.$$

Here $|1\rangle$ is the excited state and $|0\rangle$ the ground state. The map sends

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \quad \mapsto \quad \begin{pmatrix} \rho_{00} + \gamma\rho_{11} & \sqrt{1-\gamma}\,\rho_{01} \\ \sqrt{1-\gamma}\,\rho_{10} & (1-\gamma)\rho_{11} \end{pmatrix}.$$

Excited-state population ρ_{11} decays to zero, transferring probability to ρ_{00}; coherences decay by a factor $\sqrt{1-\gamma}$. In the continuous-time limit, $\rho_{11}(t) = \rho_{11}(0)e^{-t/T_1}$, where T_1 is the **relaxation time**.

Generalized amplitude damping

At finite temperature, excited states can be *excited* back from the environment. The generalized channel sends ρ_{11} toward the thermal equilibrium population p_∞ rather than to zero, with Kraus operators parameterized by both γ and p_∞. The ground state eventually becomes a Gibbs-weighted mixture.

Intuition

Every realistic noise source on a quantum system can be schematically split into:

- **Phase noise (T_2 processes).** The environment "reads" which energy eigenstate the system is in, without exchanging energy. This looks like a weak Z-measurement: off-diagonal coherences decay, populations are preserved. The information that leaks to the environment is *which basis state* the system is in; the information that is lost from the system is the *relative phase* between basis states.
- **Amplitude noise (T_1 processes).** The environment exchanges energy with the system, pulling the population from the excited state to the ground state (or toward thermal equilibrium). This is what you get from spontaneous emission into a zero-temperature photon bath, or from any dissipative coupling.

Both happen simultaneously in real systems, and the dominant one depends on the platform: superconducting qubits are typically T_1-dominated (dielectric loss, radiative emission into the resonator), trapped-ion qubits are typically T_2-dominated (magnetic-field fluctuations, laser phase noise), photonic qubits are dominated by photon loss (a specific form of amplitude damping).

The two processes have very different structural character:

- **Dephasing** is *reversible* in principle — it is caused by the environment coupling to a diagonal observable, and if the environment's state were recorded and fed back the coherences could in principle be recovered. It is the quantum analogue of pure "measurement back-action without readout."
- **Amplitude damping** is *irreversible* — energy has actually left the system and entered the environment. Undoing it would require reversing the energy flow, which is an exact time-reversal of the environment's dynamics.

The distinction between the two is the structural content of "$T_2 \neq T_1$" — you can have phase noise without energy loss, but you cannot have energy loss without phase noise.

Structural Role

Dephasing and amplitude damping are the load-bearing noise primitives of quantum information:

- **Noise models for VQA and NISQ.** Circuit-level noise is typically modeled as single-qubit dephasing plus amplitude damping between gates, with two-qubit gate-specific error channels added on top. Benchmarking and error mitigation protocols target these two channels in particular.
- **Lindblad generators.** The continuous-time limits of these channels give the Lindblad dissipators $\mathcal{L}_Z$ (dephasing) and $\mathcal{L}_{\sigma_-}$ (relaxation) of Module 8.
- **Coherence time budgeting.** T_1 and T_2 are the headline hardware figures of merit: the number of coherent gates that can be executed before noise dominates is $\sim \min(T_1, T_2)/\tau_g$, where τ_g is the gate time.

- **$T_2 \le 2T_1$.** A general relationship: pure dephasing is always bounded below by the dephasing induced by relaxation. Any T_1 process contributes to T_2, so the pure dephasing rate is $1/T_\phi = 1/T_2 - 1/(2T_1) \ge 0$.
- **Error correction (Module 7).** Stabilizer codes target the bit-flip, phase-flip, and combined errors that these channels produce in the Pauli decomposition.
- **Bloch ball picture.** Dephasing collapses the Bloch ball onto the z-axis; amplitude damping collapses it onto the south pole (ground state). The geometric picture is cleanest for a qubit.

Mathematical Development

Unitary dilation of the dephasing channel

The dephasing channel $\Phi_{\text{deph}}(\rho) = (1-p)\rho + pZ\rho Z$ arises from a CNOT-like coupling to an ancilla qubit:

$$U = |0\rangle\langle 0|_S \otimes I_E + |1\rangle\langle 1|_S \otimes X_E,$$

applied to a system state $\rho_S \otimes |e_0\rangle\langle e_0|$ with ancilla initial state $|e_0\rangle = \sqrt{1-p/2}|0\rangle_E + \sqrt{p/2}|1\rangle_E$. Tracing out the ancilla gives the dephasing channel. This is a direct instance of Stinespring dilation: a concrete joint unitary + partial trace realization of a noise channel.

Unitary dilation of the amplitude damping channel

The amplitude damping channel arises from a beam-splitter-like coupling between system and environment (initially in the ground state):

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-\gamma} & \sqrt{\gamma} & 0 \\ 0 & -\sqrt{\gamma} & \sqrt{1-\gamma} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

acting on $(|0\rangle, |1\rangle)_S \otimes (|0\rangle, |1\rangle)_E$ with initial environment state $|0\rangle_E$. Tracing out the environment recovers the Kraus operators above. Physically: if the system is in the excited state $|1\rangle$, it transfers a quantum of energy to the environment with probability γ, leaving both in their respective ground/excited states entangled.

Bloch ball dynamics

Under dephasing with parameter p, the Bloch vector (r_x, r_y, r_z) evolves as

$$(r_x, r_y, r_z) \mapsto ((1-2p)r_x, (1-2p)r_y, r_z).$$

The ball is compressed along the xy-plane, collapsing onto the z-axis as $p \to 1/2$. The z-axis is the *decoherence-free* direction: eigenstates of Z are fixed points.

Under amplitude damping with parameter γ,

$$r_x \mapsto \sqrt{1-\gamma}\, r_x, \quad r_y \mapsto \sqrt{1-\gamma}\, r_y, \quad r_z \mapsto (1-\gamma)r_z + \gamma.$$

The ball is compressed and shifted: xy-components decay by $\sqrt{1-\gamma}$, z-component decays toward the ground state $r_z = 1$ (the fixed point is the south pole if we orient $|0\rangle$ downward, or the north pole if upward). As $\gamma \to 1$ every state is sent to the ground state $|0\rangle$.

The $T_2 \le 2T_1$ bound

Pure dephasing has rate $1/T_\phi$; amplitude damping has rate $1/T_1$. The total coherence decay rate is

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}.$$

The factor of $1/2$ in the T_1 contribution is because amplitude damping decays the coherences at *half* the rate at which it decays the populations (this follows from the Kraus operator K_0, which has $\sqrt{1-\gamma}$ on the excited entry but no decay on the ground entry). Hence $T_2 \le 2T_1$, with equality when pure dephasing vanishes. In practice, pure dephasing is usually significant, so $T_2 < 2T_1$.

Composition of channels

The product of dephasing and amplitude damping is computed by composing Kraus operators. The result is another CPTP channel with six Kraus operators (the tensor-product structure), describing combined noise. In the continuous-time Lindblad limit, dephasing and amplitude damping combine additively at the level of dissipators:

$$\mathcal{L}(\rho) = \gamma_\phi\big(Z\rho Z - \rho\big) + \gamma_1\big(\sigma_-\rho\sigma_+ - \tfrac{1}{2}\{\sigma_+\sigma_-, \rho\}\big).$$

The two generators commute only if γ_ϕ or γ_1 vanishes; otherwise the dynamics must be solved jointly.

Worked Example

Qubit Bloch ball under dephasing

Start with a pure state on the equator of the Bloch sphere, $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, density matrix

$$|+\rangle\langle+| = \tfrac{1}{2}(I + X) = \tfrac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

Bloch vector $\vec{r} = (1, 0, 0)$, pure (length 1).

Apply dephasing at rate $1/T_2$ for time t. The off-diagonal element shrinks by e^{-t/T_2}:

$$\rho(t) = \tfrac{1}{2}\begin{pmatrix} 1 & e^{-t/T_2} \\ e^{-t/T_2} & 1 \end{pmatrix}.$$

Bloch vector $\vec{r}(t) = (e^{-t/T_2}, 0, 0)$. At $t \to \infty$, $\rho \to I/2$, maximally mixed. Entropy grows from 0 to 1 bit; purity decreases from 1 to 1/2. The state migrates along the x-axis from the Bloch sphere to the center.

Crucially, the state $|0\rangle$ (Bloch vector $(0, 0, 1)$) is completely unchanged by dephasing — it is a pointer state of the Z-dephasing channel. Any classical mixture of $|0\rangle, |1\rangle$ is also unchanged.

Excited state under amplitude damping

Start with $\rho(0) = |1\rangle\langle 1|$, the excited state. Under amplitude damping with rate $1/T_1$,

$$\rho_{11}(t) = e^{-t/T_1}, \qquad \rho_{00}(t) = 1 - e^{-t/T_1}.$$

The excited-state population decays exponentially to zero, and the lost probability goes entirely to the ground state. At $t \to \infty$, $\rho \to |0\rangle\langle 0|$, the pure ground state. Purity is preserved at infinity (the fixed point is pure), even though the trajectory passes through mixed states at intermediate times.

For an initial superposition $|+\rangle$, amplitude damping drives the state toward the ground state $|0\rangle$, not the equatorial center $I/2$ — a key difference from dephasing. The asymptotic state is pure because zero-temperature amplitude damping has a pure fixed point.

Combined dynamics for superconducting transmon qubit

For a typical superconducting qubit: $T_1 \approx 100\,\mu s$, $T_2 \approx 50\,\mu s$, gate time $\tau_g \approx 10$ ns. The Bloch vector at time t starting from $|+\rangle$:

$$\vec{r}(t) = \left(e^{-t/T_2}, 0, 1 - e^{-t/(2T_1)}\right) \quad \text{(approximately)}.$$

In the long-time limit the state converges to $(0, 0, 1) = |0\rangle$, the ground state. The decoherence time T_2 sets the timescale for the xy-plane components; the relaxation time T_1 sets the timescale for the z-component's approach to the ground state. The ratio $T_2/\tau_g \approx 5000$ gives a rough upper bound on circuit depth before noise dominates.

Cross-Links

- **Module 5.3 (System–environment coupling):** Dephasing and amplitude damping are the reduced dynamics for specific choices of H_{SE} (diagonal vs. off-diagonal coupling).
- **Module 5.5 (Pointer basis):** Dephasing makes the Z-basis the pointer basis. Amplitude damping makes the ground state the unique fixed point.
- **Module 5.6 (Decoherence timescales):** T_1 and T_2 parameterize these channels in continuous-time form.
- **Module 8.2 (Lindblad equation):** The continuous-time generators $\mathcal{L}_Z$ and $\mathcal{L}_{\sigma_-}$ are the infinitesimal versions of these channels.
- **Module 7 (Error correction):** Stabilizer codes are designed to correct the Pauli decomposition of these noise channels.
- **VQA:** Circuit-level noise is typically modeled as compositions of these two primitives.

Structural Compression

Invariant: Dephasing destroys phase information and preserves energy; amplitude damping destroys excitation and exchanges energy.

Mechanism: Two Lindblad generators: $\mathcal{L}_Z(\rho) = Z\rho Z - \rho$ (dephasing) and $\mathcal{L}_{\sigma_-}(\rho) = \sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\}$ (relaxation).

Cross-System Echo: Friction in classical mechanics dissipates kinetic energy; phase noise in classical oscillators shifts relative phases without changing amplitude. The two are quantum-mechanically the same distinction made precise: T_1 = classical friction, T_2 = classical phase noise, with the additional quantum constraint $T_2 \leq 2T_1$ that has no classical analogue.

Exercises

1. Verify that the dephasing Kraus operators $K_0 = \sqrt{1 - p/2}I, K_1 = \sqrt{p/2}Z$ satisfy $K_0^\dagger K_0 + K_1^\dagger K_1 = I$.
2. Verify that the amplitude damping Kraus operators satisfy completeness and compute their action on the Bloch vector.
3. Show that the dephasing channel is unital (preserves I), but the amplitude damping channel is not.
4. Compute the composition of dephasing with parameter p_1 and dephasing with parameter p_2. Verify it is dephasing with parameter $p_1 + p_2 - 2p_1p_2$.

5. Show that amplitude damping has a unique fixed point $|0\rangle\langle 0|$, while dephasing has a one-parameter family of fixed points (all Z-diagonal states).
6. Derive the $T_2 \leq 2T_1$ bound by computing how amplitude damping alone affects the off-diagonal elements of ρ and comparing with the population decay rate.
7. Argue, structurally, why no single decoherence channel can be both energy-conserving *and* produce a pure-state fixed point other than an energy eigenstate.

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Concepts: quantum-channels, quantum-noise, decoherence
Prerequisites: 5.3
Enables: 5.5

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