Decoherence Timescales

Quantum Foundations · Module 5 — Decoherence

Node 5.6

Position in Knowledge Graph

System: Quantum Foundations **Structural Domain:** Decoherence **Node:** 5.6

How fast does decoherence happen? For macroscopic objects the answer is "unobservably fast" — Schrödinger's cat decoheres in something like 10^{-23} seconds. For microscopic systems the answer is "fast enough to matter but slow enough to exploit" — superconducting qubits maintain coherence for microseconds to milliseconds, long enough to run quantum circuits. The structural scaling that explains both regimes, and everything in between, is the topic of this node. Decoherence rates depend on the coupling strength, the spectral density of the environment at the relevant frequencies, and the "size" of the superposition in whatever basis is being einselected. Understanding this scaling is the key to both the apparent classicality of the macroscopic world and the engineering challenge of building quantum computers.

Formal Definition

T_1, T_2, T_ϕ

For a qubit coupled to an environment, the decoherence times are defined operationally from the decay of density-matrix elements in the energy eigenbasis:

- **Relaxation time T_1:** The characteristic timescale of population decay. If the qubit starts in the excited state, $\rho_{11}(t) - \rho_{11}^{(eq)} = (\rho_{11}(0) - \rho_{11}^{(eq)})e^{-t/T_1}$, where $\rho_{11}^{(eq)}$ is the thermal equilibrium excited-state population.
- **Dephasing time T_2:** The characteristic timescale of off-diagonal coherence decay. $|\rho_{01}(t)| = |\rho_{01}(0)|e^{-t/T_2}$.
- **Pure dephasing time T_ϕ:** The coherence decay due to phase noise alone, not counting the contribution from relaxation. Related by

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}.$$

The factor of 1/2 in the T_1 contribution reflects the fact that amplitude damping decays off-diagonal elements at half the rate it decays populations (the Kraus operator $K_0 = \mathrm{diag}(1, \sqrt{1-\gamma})$ gives $\sqrt{1-\gamma}$ on the off-diagonal but $(1-\gamma)$ on the population).

General decoherence rate from Fermi's golden rule

For a system weakly coupled to a bath through $H_{SE} = A \otimes B$ (single system operator A, single bath operator B), the decoherence rate at second order in perturbation theory is

$$\Gamma = \frac{1}{\hbar^2}\int_{-\infty}^{\infty} d\tau\, \langle B(\tau)B(0)\rangle_{\rho_E}\, e^{i\omega\tau} = \frac{1}{\hbar^2}\, S_B(\omega),$$

where $S_B(\omega)$ is the **spectral density** of the bath operator B at the transition frequency ω corresponding to the system process of interest. This is Fermi's golden rule for decoherence: the rate is the Fourier transform of the bath's autocorrelation function evaluated at the system's characteristic frequency.

For the qubit relaxation rate,

$$\frac{1}{T_1} = \frac{|g|^2}{\hbar^2} J(\omega_0) \left[n_{\text{th}}(\omega_0) + 1 \right],$$

where g is the coupling strength, $J(\omega)$ is the environment's spectral density, ω_0 is the qubit transition frequency, and $n_{\text{th}}(\omega) = (e^{\beta\omega} - 1)^{-1}$ is the thermal occupation.

For the pure dephasing rate at low frequencies,

$$\frac{1}{T_\phi} = \frac{|g|^2}{\hbar^2} S_z(0),$$

where $S_z(0)$ is the zero-frequency spectral density of the environmental longitudinal noise. Dephasing is sensitive to *low-frequency* noise (flicker noise, $1/f$ noise) while relaxation is sensitive to noise at the qubit frequency.

Decoherence rate for macroscopic superpositions

For a massive object in a spatial superposition of separation Δx, scattering off environmental particles of thermal wavelength λ, the decoherence rate is (Joos–Zeh 1985, Gallis–Fleming 1990)

$$\Gamma \sim \Lambda \, (\Delta x)^2,$$

with Λ a scattering constant that scales linearly with the environmental density and scattering cross-section. The $(\Delta x)^2$ scaling is the structural signature: **decoherence rate grows as the square of the superposition size**. For any macroscopic Δx, this gives astronomically large rates.

Intuition

Three structural factors determine how fast a system decoheres:

1. **Coupling strength.** Stronger coupling to the environment means faster decoherence. The rate is typically quadratic in the coupling, $\Gamma \propto |g|^2$.
2. **Bath spectral density.** The environment must have modes at the relevant frequency to absorb the system's information. A bath with no modes at the system's transition frequency cannot drive relaxation, no matter how strong the coupling.
3. **Superposition "size".** For a spatially extended superposition, the decoherence rate grows with the square of the separation, because more environmental scattering events are needed to resolve a larger superposition. This is the reason Schrödinger's cat decoheres so fast: its superposition covers macroscopic distances.

The combined message: **the more macroscopic, the more coupled, and the noisier the environment, the faster decoherence.** This is a quantitative gradient, not a sharp boundary — smaller, better-isolated, colder systems decohere slower, and the line between "quantum" and "classical" is determined by where the decoherence rate crosses the threshold of observational irrelevance.

The key intuitive distinction between T_1 and T_2:

- T_1 **(relaxation)**: requires *energy exchange* with the environment. The bath must have modes at the qubit transition frequency to accept a quantum of energy. Typically limited by the density of states and temperature of the environment at ω_0.
- T_ϕ **(pure dephasing)**: requires only *phase fluctuations* of the qubit's energy levels. No energy exchange, so any low-frequency noise (temperature fluctuations, flux noise, charge noise) contributes. Usually dominated by $1/f$ noise in solid-state qubits.

In most experimental platforms, pure dephasing dominates: $T_\phi \ll 2T_1$, so $T_2 \approx T_\phi$. Exceptions are systems with good shielding from low-frequency noise (trapped ions in magnetic shielding, NV centres in isotopically purified diamond), where $T_2 \approx 2T_1$ can be approached.

Structural Role

Decoherence timescales are the load-bearing hardware parameters of every quantum technology:

- **Quantum computing depth budget.** The number of gates that can be executed coherently is roughly $N \sim T_2/\tau_g$, where τ_g is the gate time. For current superconducting qubits, $T_2 \sim 50\mu s$ and $\tau_g \sim 10$ ns, giving $N \sim 5000$. Increasing either parameter is a major engineering target.
- **Error correction threshold.** Fault-tolerant quantum computing requires physical error rates below $\sim 10^{-3}$ per gate. For gates with time τ_g, this means $T_2/\tau_g \gtrsim 1000$. Reaching this threshold is the structural gate to scalable quantum computing.
- **Quantum sensing and metrology.** The Ramsey interferometry uncertainty on a frequency measurement scales as $1/(T_2\sqrt{N})$ where N is the number of shots. Longer T_2 means better sensors.
- **The classical limit.** For macroscopic objects, $\Gamma \sim 10^{23}\,\mathrm{s}^{-1}$ or larger. This is the structural reason "classicality" is the overwhelming norm: the decoherence rate is faster than any physical timescale. Quantum coherence is the exception, requiring deliberate isolation.
- **Quantum Zeno effect.** When the environment couples strongly enough, $T_2 \ll$ any internal dynamical time, the system is pinned in the pointer basis by continuous einselection. This is the structural mechanism behind the Zeno effect (Node 3.5/3.6 continuous limit).
- **Hardware platform comparison.** Different platforms have wildly different T_1, T_2 profiles. Trapped ions: $T_1 \sim$ hours, $T_2 \sim$ seconds. Superconducting qubits: $T_1 \sim 100\mu s$, $T_2 \sim 50\mu s$. Photonic qubits: no relaxation (photons don't decay), but photon loss (amplitude damping) over fiber is the effective coherence limit. NV centres: $T_2 \sim$ ms at room temperature, seconds at cryogenic.

Mathematical Development

Derivation of Fermi's golden rule for decoherence

Consider $H = H_S + H_E + g\, A_S \otimes B_E$ and a system initially in $|\psi_i\rangle$ and environment in ρ_E (thermal state of H_E). At second order in g, the rate of transition out of $|\psi_i\rangle$ is

$$\Gamma_{i\to f} = \frac{2\pi}{\hbar}|g|^2|\langle f|A_S|i\rangle|^2\,\delta(E_f - E_i - \omega)\,S_B(\omega),$$

summed over final states $|f\rangle$. The transition rate depends on the matrix elements of the system operator, the density of final states, and the bath spectral density at the right frequency. This is the standard Fermi's golden rule for transitions into a continuum.

For decoherence specifically, the coherence between two states $|i\rangle, |j\rangle$ decays at rate

$$\frac{1}{T_{ij}} = \frac{1}{2}\left(\frac{1}{T_1^{(i)}} + \frac{1}{T_1^{(j)}}\right) + \frac{1}{T_\phi^{(ij)}},$$

where the first term is the average of the two states' relaxation rates (amplitude damping) and the second is pure dephasing between the pair. For a two-level system this reduces to the standard $1/T_2 = 1/(2T_1) + 1/T_\phi$.

Short-time vs. long-time behavior

At very short times, decoherence is *quadratic* in time rather than exponential:

$$|\rho_{01}(t)| \approx |\rho_{01}(0)|\,(1 - \tfrac{1}{2}\gamma_0 t^2 + O(t^4)),$$

with $\gamma_0 = \mathrm{Var}(H_{SE})/\hbar^2$ the variance of the coupling. This is the **Zeno regime**. As t increases past the bath correlation time τ_c, the dynamics transition to exponential decay:

$$|\rho_{01}(t)| \sim e^{-t/T_2}.$$

The transition happens at $t \sim \tau_c$, the inverse of the environment's characteristic frequency cutoff. For $t \ll \tau_c$, frequent measurement (or even the Zeno-type self-interruption) can suppress decoherence; for $t \gg \tau_c$, exponential decay is unavoidable.

Scaling with system size

For a collection of N qubits coupled independently to the same environment, the *collective* decoherence rate can scale as N^2 rather than N — **superdecoherence**. This is the structural reason that decoherence-free subspaces (which are immune to collective noise) become increasingly valuable at large N: they are the only way to resist the superdecoherence scaling.

For a spatial superposition of extent Δx involving many constituents, the decoherence rate is bounded below by the per-constituent rate times $(\Delta x/\lambda)^2$ (with λ the environmental thermal wavelength). For a cat of mass M in a superposition of macroscopic separation, the product gives $\Gamma \propto M(\Delta x)^2$, rendering coherent superpositions of classical objects unobservable.

Spectral density and noise types

Different noise spectra give different scaling of T_2 with measurement time:

- **White noise** (flat $S(\omega)$): exponential decay, well-defined T_2.
- **$1/f$ noise** (ubiquitous in solid-state systems): $|\rho_{01}(t)| \sim e^{-t^2/T_\phi^2} \cdot$ corrections — Gaussian decay, no fixed T_2 but an effective one at a given timescale.
- **Ohmic spectral density** $J(\omega) \propto \omega$ at low frequency: exponential decay at finite temperature, power-law at zero temperature.

The structural consequence: the figure of merit T_2 is a simplification that hides the true nature of the noise. Realistic systems show non-exponential decay, and the effective T_2 depends on which dynamical range is being probed. This is why dynamical decoupling sequences (CPMG, UDD) can *extend* apparent T_2 — they filter out certain frequency ranges of noise that would otherwise contribute.

Worked Example

Superconducting transmon qubit

Typical transmon parameters (as of 2025):

- Qubit frequency $\omega_0/2\pi \approx 5$ GHz
- $T_1 \approx 100\,\mu s$ (limited by dielectric loss and Purcell decay into the readout resonator)
- $T_2 \approx 50\,\mu s$ (limited by flux noise and charge noise at low frequencies)
- Single-qubit gate time $\tau_g \approx 20$ ns

- Two-qubit gate time $\tau_{2g} \approx 200$ ns

Gates per T_2: $T_2/\tau_g \approx 2500$. **Circuits of depth 1000** are possible with some error; deeper circuits require error correction.

Threshold for fault tolerance: Surface code threshold is approximately $p_{\text{th}} \sim 10^{-2}$ for circuit-level noise. This requires $T_2/\tau_g \gtrsim 100$, easily satisfied. The harder challenge is two-qubit gate fidelity, which must be $> 99\%$ and currently hovers at 99.5–99.9% in the best systems.

Macroscopic Schrödinger's cat

Following Zurek's estimate: take a 1 kg cat in a superposition of two positions 10 cm apart, embedded in a room-temperature gas bath. The relevant parameters:

- Molecular scattering cross-section $\sigma \sim 10^{-19}\,\text{m}^2$
- Gas density $n \sim 10^{25}\,\text{m}^{-3}$
- Thermal velocity $v \sim 500\,\text{m/s}$
- Thermal de Broglie wavelength $\lambda \sim 10^{-11}\,\text{m}$
- Superposition extent $\Delta x \sim 10^{-1}\,\text{m}$

Decoherence rate:

$$\Gamma \sim n\sigma v\,\frac{(\Delta x)^2}{\lambda^2} \sim 10^{25} \cdot 10^{-19} \cdot 10^2 \cdot 10^{20} \approx 10^{28}\,\text{s}^{-1}.$$

This gives $\tau_D \sim 10^{-28}$ seconds. The spatial superposition is coherent for less than the Planck time. It is unobservable in principle, not just in practice.

Nuclear spin qubit

For a ^{31}P nuclear spin in silicon (Kane qubit concept), the relevant coupling is the hyperfine interaction with the bath of ^{29}Si nuclei. Isotopically purified ^{28}Si reduces the bath. Measured values (Tyryshkin et al. 2012):

- $T_1 \sim$ hours (nuclear spin relaxation is very slow — nuclear magnetic moments are tiny)
- $T_2 \sim 30$ seconds (dynamical decoupling extends this further)

These are among the longest coherence times ever measured for solid-state qubits. The structural reason is that nuclear spins couple weakly to everything: the hyperfine coupling is small, the gyromagnetic ratio is $1000\times$ smaller than for electrons, and the relevant noise frequencies are far from thermal fluctuations. This is a design-principle illustration of "minimize coupling + minimize spectral density at relevant frequencies."

Cross-Links

- **Module 5.4 (Decoherence mechanisms):** T_1 and T_2 parameterize the canonical dephasing and amplitude damping channels.
- **Module 5.5 (Pointer basis):** The decoherence time sets the rate at which the reduced state becomes diagonal in the pointer basis.
- **Module 8.2 (Lindblad equation):** The continuous-time Lindblad dynamics have T_1^{-1} and T_2^{-1} as the dissipator rates.
- **VQA:** Coherence time sets the circuit depth budget. Barren plateaus and trainability depend on whether the circuit fits within the coherence window.
- **Module 10.4 (Quantum-to-classical transition):** The structural fact that decoherence is fast explains the emergence of classicality.
- **Quantum sensing:** Metrological sensitivity scales with T_2; longer coherence means better measurements.

Structural Compression

Invariant: Decoherence rate scales with the *squared coupling strength*, the *spectral density of the environment at the relevant frequency*, and (for spatial superpositions) the *squared separation*.

Mechanism:

$$\Gamma \sim \frac{|g|^2}{\hbar^2} S_B(\omega) \quad \text{(Fermi's golden rule for a local coupling)},$$

with S_B the bath spectral density and ω the system's characteristic frequency.

Cross-System Echo: Damping rates in classical oscillators follow the same structural form — coupling squared times spectral density at the resonance frequency. The distinctive quantum feature is the $(\Delta x)^2$ scaling for spatial superpositions, which has no classical analogue and is why macroscopic objects are "classical" to overwhelming precision: the decoherence rate is driven catastrophically high by the macroscopic size of any would-be superposition.

Exercises

1. Verify the relation $1/T_2 = 1/(2T_1) + 1/T_\phi$ by computing how the amplitude damping channel affects the off-diagonal element of a qubit density matrix.
2. For a qubit with $T_1 = 100\,\mu s$ and pure dephasing $T_\phi = 70\,\mu s$, compute T_2.
3. Derive the $(\Delta x)^2$ scaling of the decoherence rate for a spatial superposition in a thermal bath, under the assumption that scattering events are localized on the thermal wavelength.
4. For an Ohmic bath with spectral density $J(\omega) = \eta\omega e^{-\omega/\omega_c}$, compute the pure dephasing rate at zero temperature and at finite temperature β.
5. Estimate the decoherence time of a single atom in a magneto-optical trap at room temperature, and compare to a trapped ion in an ultra-high-vacuum chamber.
6. For a transmon qubit with $T_1 = 100\,\mu s$ and gate time 20 ns, compute the gate error rate if all error is coherence-limited, and assess whether fault tolerance thresholds are met.
7. Argue, structurally, why macroscopic objects cannot exhibit coherent quantum behavior under realistic environmental conditions, and quantify the "macroscopicity" at which decoherence becomes faster than any observational timescale.

Quantum Foundations · Module 5 — Decoherence · Node 5.6
Concepts: decoherence, quantum-noise
Prerequisites: 5.4

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