Module 6 — Interpretations

Quantum Foundations

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The Measurement Problem (Frame Node)

Position in Knowledge Graph

System: Quantum Foundations **Structural Domain:** Interpretations **Node:** 6.1

This is the frame node for the entire interpretations module. Every interpretation discussed in subsequent nodes is, structurally, a different proposed resolution of the same problem: the formal incompatibility between unitary evolution (Node 2.4) and the projection postulate (Node 2.3). The HBAR Method requires that we name this incompatibility precisely before introducing any interpretive framework.

Formal Statement

The standard formulation of quantum mechanics contains two distinct dynamical rules:

1. **Unitary evolution** (Postulate, Node 2.4): when no measurement occurs, the state $|\psi\rangle$ evolves smoothly and reversibly under the Schrödinger equation

   $$i\hbar\frac{d}{dt}|\psi\rangle = H|\psi\rangle, \qquad |\psi(t)\rangle = U(t)|\psi(0)\rangle.$$

   This map is linear, deterministic, norm-preserving, and reversible.

2. **Projection postulate / collapse** (Born rule, Node 2.3): when a measurement of an observable $A = \sum_i \lambda_i P_i$ occurs, the state jumps:

   $$|\psi\rangle \;\longrightarrow\; \frac{P_i|\psi\rangle}{\sqrt{\langle\psi|P_i|\psi\rangle}}$$

   with probability $p(i) = \langle\psi|P_i|\psi\rangle$. This map is nonlinear, stochastic, non-norm-preserving on individual branches, and irreversible.

The **measurement problem** is the structural fact that the formalism does not specify when, where, or under what conditions rule 2 supplants rule 1.

Why This Is a Structural Problem and Not a Philosophical One

The measurement problem is *not* the question "what is consciousness?" or "what does measurement mean?" These are downstream concerns. The structural problem is internal to the formalism:

- A measurement apparatus is itself a physical system, hence its dynamics should be unitary (rule 1).
- The system + apparatus together are a composite quantum system (Node 2.5), evolving under a joint Hamiltonian.

- A faithful unitary description of measurement produces an *entangled* system-apparatus state, not a collapsed one.
- Yet measurement statistics are observed to follow rule 2, not the unitary prediction.

So either:

(a) Rule 1 does not apply at the macroscopic level (some new physics intervenes — e.g., spontaneous collapse).
(b) Rule 2 is not fundamental but emerges from rule 1 plus some additional structural ingredient (decoherence, branching, hidden variables, agent-relative states).
(c) The two rules describe complementary aspects of the same physical situation under different epistemic frames (Copenhagen, QBism).

Each option in the rest of this module is one of these branches. None has been ruled out by experiment.

Intuition

The simplest framing: write down the unitary evolution of "system + measurement apparatus + observer" under the Schrödinger equation, all the way to the end. Where in this calculation does the wavefunction collapse? The honest answer is: nowhere. The Schrödinger equation is linear, so superpositions of (system, apparatus, observer) states evolve into superpositions of (system, apparatus, observer) states. Collapse is added by hand at some point — and the formalism does not say where.

Structural Role

This frame node organizes Module 6. Each subsequent node presents one interpretive resolution and is judged on three criteria:

1. **Formal completeness:** Does it specify exactly when (or whether) collapse occurs?
2. **Empirical equivalence:** Does it reproduce the standard predictions of quantum mechanics?
3. **Ontological commitments:** What does it claim *exists* — wavefunctions, branches, hidden variables, agents?

The HBAR Method's "no interpretational rhetoric without formal grounding" rule (course-summary §1) is enforced here. We do not argue about meaning until we have stated the formal structure.

Mathematical Development

The Wigner's-friend setup (canonical illustration)

Let S be a qubit, M be an apparatus modeled as a qubit, and F (Wigner's friend) be a third qubit. The interaction:

1. Initial state: $(|0\rangle_S + |1\rangle_S)|0\rangle_M|0\rangle_F$.
2. SM interaction: CNOT from S to M, entangling them.
3. MF interaction: CNOT from M to F, entangling F with the joint SM state.

The final state, by purely unitary evolution, is

$$\frac{1}{\sqrt{2}}(|0\rangle_S|0\rangle_M|0\rangle_F + |1\rangle_S|1\rangle_M|1\rangle_F).$$

The friend (and the apparatus, and the system) are all in a superposition. Where did collapse happen?

According to:

- **Copenhagen:** Collapse occurred when "the friend looked." Why? Unspecified.
- **Many-worlds:** It didn't. Each branch is a separate world.
- **Bohmian mechanics:** Particles always had definite positions; the wavefunction guides them.
- **QBism:** "Collapse" is the friend updating their personal probabilities; nothing physical happened to anyone else.
- **Spontaneous collapse:** A physical collapse process occurs at some hidden mass/complexity threshold; testable.

Each is a different commitment about what is actually going on inside the unitary expression above.

Worked Example

Schrödinger's cat as macroscopic Wigner's friend

Take a radioactive atom with a half-life $t_{1/2}$ coupled to a Geiger counter, which triggers a vial of poison, which kills a cat in a sealed box. The atom's state after time $t_{1/2}$ is

$$|\psi_{\text{atom}}\rangle = \tfrac{1}{\sqrt{2}}(|\text{undecayed}\rangle + |\text{decayed}\rangle).$$

Unitary evolution of the full system. Atom, counter, vial, and cat are all quantum systems. The Schrödinger equation applied to the joint system gives

$$|\Psi\rangle = \tfrac{1}{\sqrt{2}}\Big(|\text{undecayed}\rangle|\text{counter quiet}\rangle|\text{vial sealed}\rangle|\text{cat alive}\rangle + |\text{decayed}\rangle|\text{counter fired}\rangle|\text{vial broken}\rangle|\text{cat dead}\rangle\Big).$$

By the linearity of Schrödinger evolution there is no step in this chain where the superposition is resolved into one branch. The final joint state is a superposition of "cat alive" and "cat dead" — a macroscopic Schrödinger cat.

Decoherence check. Tracing out the environment (air molecules colliding with the cat's fur, thermal photons emitted by the cat's body, etc.) diagonalizes the reduced density matrix of the cat in the pointer basis $\{|\text{alive}\rangle, |\text{dead}\rangle\}$ on timescales of $\sim 10^{-23}$ seconds (10.4). After decoherence,

$$\rho_{\text{cat}} \approx \tfrac{1}{2}|\text{alive}\rangle\langle\text{alive}| + \tfrac{1}{2}|\text{dead}\rangle\langle\text{dead}|.$$

This is a classical probabilistic mixture — no interference, no off-diagonal coherences. Decoherence has solved the "why no cat-interference-experiment" problem. But ρ_{cat} is still a mixture: the formalism says "probability 1/2 alive, probability 1/2 dead," not "alive" or "dead." In a single run, an observer opens the box and sees exactly one outcome.

What each interpretation says happens when Alice opens the box:

- **Copenhagen.** Before opening: system is in a superposition; talking about "the cat" as if it has a definite state is meaningless. On opening: the wavefunction collapses (non-unitarily) to one of the two branches. The cat "becomes" alive or dead at the moment of observation. No account of *when* this happens; the classical/quantum cut is placed wherever is convenient.
- **Many-worlds.** Before opening: the universe is in a superposition. On opening: Alice becomes entangled with the cat, producing $\frac{1}{\sqrt{2}}(|\text{alive}\rangle|\text{Alice sees alive}\rangle + |\text{dead}\rangle|\text{Alice sees dead}\rangle)$. Both branches are equally real. Each branch contains a version of Alice who is certain she saw a single outcome. Nothing is selected; nothing collapses.

- **Bohmian mechanics.** Before opening: the cat has a definite configuration (all particle positions are specified) — either the "alive" configuration or the "dead" configuration, determined by the initial hidden variable and guided by the pilot wave. The wavefunction is still a superposition of both, but only one is occupied by the actual particles. Opening reveals pre-existing fact.
- **QBism.** Before opening: Alice has a personal probability assignment of 1/2 for each outcome. On opening: Alice updates her beliefs. "Collapse" is a Bayesian update on her personal credences; no physical collapse occurred and the wavefunction was never a physical object.
- **GRW / dynamical collapse.** Before opening: a macroscopic object like a cat is far above the spontaneous-collapse threshold. Within microseconds, the cat's wavefunction has already physically collapsed to either $|\text{alive}\rangle$ or $|\text{dead}\rangle$ via spontaneous localizations — well before Alice arrives. The "opening" is just classical inspection of an already-definite situation.

The structural point. All five interpretations agree on every *observable prediction* in this experiment: Alice sees alive with probability 1/2 and dead with probability 1/2. The disagreement is entirely about the ontological content of the intermediate state and about what "happens" during the act of observation. This is why the measurement problem is simultaneously the most empirically stable and the most conceptually contested problem in the foundations of physics.

Cross-Links

- **Module 2:** Postulates 2.3 (collapse) and 2.4 (unitary) — the two rules whose tension defines the problem.
- **Module 5:** Decoherence (Node 5.5) — does *not* solve the measurement problem (it explains the apparent classicality of pointer states but leaves the choice of branch unaccounted).
- **Module 10:** PBR theorem (10.3) and other foundational no-go results constrain admissible interpretations.

Structural Compression

Invariant: The formalism contains two distinct dynamical rules and no internal criterion specifying when each applies.

Mechanism: Linearity of Schrödinger evolution combined with nonlinearity of the projection postulate.

Cross-System Echo: Frequentist vs Bayesian interpretations of probability share an analogous structural pattern: a single mathematical formalism, multiple compatible philosophical readings, no internal criterion that selects between them.

Exercises

1. State precisely the contradiction between unitary evolution and the projection postulate, using the Wigner's-friend setup.
2. Argue why decoherence alone cannot solve the measurement problem. (Hint: decoherence diagonalizes the reduced density matrix in the pointer basis but does not select a *single* outcome.)
3. For each of the five resolutions above, state the answer to: "What physically happens when Wigner's friend looks at the qubit?"
4. Argue why the measurement problem is a structural problem about the postulates, not a metaphysical question about consciousness.

Copenhagen Interpretation

Position in Knowledge Graph

System: Quantum Foundations **Structural Domain:** Interpretations **Node:** 6.2

Copenhagen is the historical default interpretation of quantum mechanics — the view Niels Bohr, Werner Heisenberg, and their colleagues developed in the late 1920s and which appears, explicitly or implicitly, in nearly every introductory textbook. It resolves the measurement problem of Node 6.1 by *postulating* a cut between quantum and classical domains and refusing to formalize where the cut lies. The wavefunction is interpreted operationally or epistemically, not as a physical field. This node states the formal content of Copenhagen precisely, separates it from the various later "Copenhagen-ish" views it is sometimes conflated with, and evaluates it against the three criteria of Node 6.1: formal completeness, empirical equivalence, and ontological commitments.

Formal Statement

The two-domain structure

Copenhagen partitions physical situations into two domains, following Bohr's framing:

1. **Quantum domain.** Systems described by a state vector $|\psi\rangle \in \mathcal{H}$ evolving under the Schrödinger equation $i\hbar\partial_t|\psi\rangle = H|\psi\rangle$. All of Nodes 2.1–2.6 applies. The state is a complete description of the system's *predictive resources*, not necessarily a description of objective facts about the system.
2. **Classical domain.** Macroscopic apparatuses, observers, and measurement outcomes. These are described in the language of classical physics — definite positions, definite pointer readings, definite events. Classical concepts are *necessary* for the formulation of experiment: one cannot specify what is being measured without using classical language to describe the apparatus.

Between the two domains sits the **Heisenberg cut**: a (movable, methodological) boundary separating quantum from classical. The location of the cut is a choice made by the physicist for the purposes of a particular analysis — it is not a structural feature of nature.

The measurement rule

When a measurement is performed, the system's state collapses according to the projection postulate of Node 2.3:

$$|\psi\rangle \mapsto \frac{P_i|\psi\rangle}{\sqrt{\langle\psi|P_i|\psi\rangle}}, \qquad p(i) = \langle\psi|P_i|\psi\rangle.$$

The collapse is *physically ambiguous* in Copenhagen. It is a rule for updating the predictive state of the quantum system conditional on a macroscopically recorded outcome, not a physical process occurring at a precise moment. The question "exactly when did the wavefunction collapse?" is declared to be meaningless: collapse is whatever is needed to make the quantum-domain description consistent with the classical-domain outcome.

Epistemic reading of the wavefunction

In Copenhagen, $|\psi\rangle$ is not a physical property of the system. It is a *calculational tool* that encodes the observer's knowledge (or the experimental setup's predictive resources) and allows the derivation of probabilities for future measurement outcomes. The question "what is the system actually doing between measurements?" is declared to be outside the scope of physics.

Different authors within the Copenhagen tradition vary on how strictly this epistemic reading is held. Bohr himself was careful not to commit to an ontological reading but emphasized the role of the classical apparatus. Heisenberg oscillated between epistemic and ontological readings. The "shut up and calculate" attitude associated with Mermin (who disavowed it) is a caricature but captures the operational spirit.

Intuition

Copenhagen is, in effect, the interpretation one arrives at by *refusing to ask* the questions that the measurement problem poses. The strategy is to draw a line at the outputs of measuring devices — classical pointer readings — and insist that physics makes predictions about these outputs via the Born rule, without committing to any story about what the wavefunction *is* or what happens *between* measurements. The epistemic reading is a natural fit: the wavefunction is whatever the physicist knows about the system, and "collapse" is just the update when new information arrives.

The appeal of Copenhagen is its operational minimalism. It requires no metaphysical commitments beyond the Born rule and the existence of classical apparatuses. It works for every actual calculation physicists do. It does not require you to believe in branching universes, hidden particles, or personal probabilities. For decades it was the interpretation physicists would give if pressed, and it is still the implicit framework of most laboratory practice.

The cost is precision: Copenhagen does not say *where* the cut lies or *what* counts as a measurement. A photomultiplier is a classical apparatus; what about a coherent superposition of photomultiplier readings? A single atom is a quantum system; what about an atom being measured by a single photon? Copenhagen's answer is that the cut can be placed anywhere that makes the analysis work — which is to say, the formalism does not determine where it should be placed. This is Bohr's **complementarity**: the placement of the cut is a feature of the observer's description, not of the physical situation.

Structural Role

Copenhagen plays a specific role within the space of interpretations:

- **The operational floor.** Whatever else an interpretation claims, it must reproduce the Born-rule predictions that Copenhagen takes as axiomatic. Every subsequent interpretation of this module accepts Copenhagen's empirical content and then attempts to provide a structural *story* behind it.
- **Resistance to the measurement problem.** Copenhagen's strategy is to declare that the measurement problem is ill-posed: there is no need to reconcile unitary evolution with collapse, because they apply in different domains. This is a *refusal* rather than a resolution, but it is internally consistent as long as the cut can always be drawn somewhere sensible.
- **The interpretive zero point.** Because Copenhagen makes no claims beyond the operational minimum, it serves as the reference against which other interpretations can be measured. Many-Worlds (6.3) removes the collapse rule; Bohmian mechanics (6.4) adds hidden variables; QBism (6.5) makes the epistemic reading explicit and agent-centric. Each modifies Copenhagen along a specific axis.
- **Tension with structural realism.** Copenhagen is strongly anti-realist about the wavefunction (or at best agnostic). This makes it compatible with Bohrian philosophical views but incompatible with any program that wants quantum mechanics to describe an observer-independent reality. Modern foundational work (Module 10) tends to treat Copenhagen as a *starting point* rather than a final resting place.

Mathematical Development

The Heisenberg cut as a boundary-condition choice

When analyzing an experiment involving a system S and an apparatus A, Copenhagen allows the Heisenberg cut to be placed anywhere along the chain from S to the final observer. Placing the cut between S and A means:

$$\rho_S = \sum_i p(i)|\phi_i\rangle\langle\phi_i|, \qquad p(i) = \text{Born rule for } S \text{ measurement.}$$

A is treated classically, with $p(i)$ as the probability of a classical pointer outcome.

Placing the cut after A means:

$$|\Psi_{SA}\rangle = \sum_i c_i|\phi_i\rangle_S|a_i\rangle_A,$$

a unitary-evolved entangled state of system and apparatus, with the cut deferred to a later stage (a second apparatus, the observer's eye, the observer's brain). Copenhagen asserts that both descriptions are operationally equivalent: they give the same predictions for whatever classical outcome is ultimately recorded.

The *mathematical* equivalence is straightforward (tracing out A in the second description gives the same statistics as the first). The *interpretive* content is that the precise location of the cut is undetermined by the formalism and must be chosen by the analyst.

Relation to decoherence

Decoherence (Module 5) provides a partial dynamical justification for the Heisenberg cut: macroscopic apparatuses decohere extremely rapidly, so their reduced density matrices become effectively diagonal in the pointer basis almost instantly. From a purely *operational* standpoint this means the apparatus's coherences cannot be detected, and so it may as well be classical.

Decoherence does *not*, however, solve the measurement problem within Copenhagen — the off-diagonal elements of the reduced density matrix become small but never actually zero, and the reduced state is still a mixed state encoding a superposition at the joint level. Copenhagen's response is that once the coherences are below detection thresholds, the pragmatic thing to do is to drop them and use a classical description. This makes the Heisenberg cut a *defensible approximation* rather than an arbitrary stipulation.

Nonetheless, from the point of view of Module 6.1's formal criterion, Copenhagen remains **formally incomplete**: it does not specify exactly when the cut should be drawn, only that it can always be drawn somewhere.

Contextuality and the observer

Bohr emphasized that the choice of experimental setup (including which classical apparatus is used) is part of the physical situation. Measuring Z and measuring X on the same qubit are not two readings of the same underlying reality — they are two *complementary* experimental contexts, each valid on its own but mutually exclusive. This view of **contextuality** anticipates the formal Kochen–Specker theorem (Node 10.2), which makes the impossibility of non-contextual hidden variable assignments rigorous. In Copenhagen, contextuality is not a bug but a feature: the classical apparatus is part of the description, and there is no observer-free quantum reality.

What Copenhagen *cannot* say

Copenhagen forbids certain questions as meaningless:

- "Which path did the particle take through the interferometer?" Meaningless, because no which-path detector was present.
- "What value did the observable have before it was measured?" Meaningless, because the question presupposes a pre-measurement value.
- "Where exactly did the wavefunction collapse?" Meaningless, because collapse is a methodological update, not a physical event.

This refusal to answer is the source of Copenhagen's critics' dissatisfaction. For operationalists, the refusal is appropriate — it marks the limits of what physics can say. For realists, it is evasive — the questions are about objective facts about nature and deserve answers.

Worked Example

Stern–Gerlach as the paradigm

The Stern–Gerlach experiment sends a beam of spin-1/2 atoms through an inhomogeneous magnetic field. Atoms with $S_z = +\hbar/2$ are deflected one way; atoms with $S_z = -\hbar/2$ are deflected the other way. A screen registers two spots.

In Copenhagen's framing:

- The **atom** (or rather, its spin) is a quantum system with state $|\psi\rangle_S$.
- The **magnet** is a classical apparatus. Its magnetic field is a classical field configuration — not a quantum field in a coherent state.
- The **detector screen** is another classical apparatus.

A single atom prepared in state $|\psi\rangle = \alpha|+\rangle + \beta|-\rangle$ passes through the apparatus. The classical description of the outcome is: with probability $|\alpha|^2$ the atom hits the upper spot, with probability $|\beta|^2$ the lower. The quantum-to-classical collapse is what happens at the detector.

The Heisenberg cut can be placed at different positions:

- **At the magnet:** The atom's spin is measured by the classical magnetic field. Outcome probabilities are computed by Born rule.
- **At the screen:** The atom+magnet system evolves unitarily into an entangled state $\alpha|+\rangle|\text{upper path}\rangle + \beta|-\rangle|\text{lower path}\rangle$, and the screen measures which path by classical detection.

Both analyses give the same predictions. Copenhagen is silent about "which one is correct" — either is acceptable as long as the computed probabilities match.

The contextual role of the magnet is essential: if instead of the z-field magnet a horizontal one were used, the apparatus would measure S_x and give different outcome probabilities. Copenhagen refuses to say that the spin "had" either an S_z or S_x value before the measurement — the apparatus choice is part of the specification of the observable.

Double-slit

A particle passes through a double-slit screen and hits a detector. Copenhagen's analysis:

- Without a which-path detector: the particle's state is in superposition across the two slits; interference pattern emerges at the detector; one cannot speak of the particle having taken a definite path.
- With a which-path detector: the particle's state is effectively measured at the slit; interference is destroyed; one of the two paths is recorded.

The classical-domain description of the experimental setup (which detectors are present) determines what can be said about the quantum-domain state. Questions like "which slit did the particle actually pass through in the interference case?" are declared meaningless by Copenhagen.

Contrast with a hidden-variable analysis

A Bohmian analysis (Node 6.4) of the same experiment would assign a definite trajectory to the particle, moving through one specific slit. Copenhagen would reject this as unnecessary metaphysical commitment: all predictions are reproduced by the standard analysis, so why commit to trajectories that are in principle unobservable?

The structural disagreement is not empirical — Copenhagen and Bohmian mechanics make the same predictions — but about what counts as a *satisfying* theory. Copenhagen says: if the predictions are right, there is nothing more to ask. Bohmian mechanics says: if predictions are all we care about, physics has abandoned its claim to describe reality.

Cross-Links

- **Module 6.1 (Measurement problem):** Copenhagen's strategy is to deny the problem is well-posed. This is internally consistent but formally incomplete.
- **Module 5.5 (Pointer basis / einselection):** Decoherence provides a dynamical story for *why* macroscopic apparatuses can be described classically, without solving the measurement problem.
- **Module 2.3 (Born rule):** Copenhagen takes the Born rule as axiomatic; other interpretations attempt to derive it.
- **Module 10.2 (Kochen–Specker):** Formalizes contextuality — Copenhagen's insistence on apparatus-dependent observables turns out to be forced, not a choice.
- **Statistics:** Frequentist statistics has a similar operational orientation — refuses to assign probabilities to hypotheses, only to observed outcomes under fixed experimental setups.

Structural Compression

Invariant: Copenhagen posits a methodological (not ontological) cut between a quantum domain of unitary dynamics and a classical domain of definite outcomes. The wavefunction is epistemic, not physical. The measurement rule is a calculational prescription, not a physical process.

Mechanism: Heisenberg cut, deliberately not formalized. Unitary evolution in the quantum domain, Born-rule probability updating at the cut, classical description of apparatuses.

Cross-System Echo: Frequentist statistics refuses to formalize the prior; Copenhagen refuses to formalize the cut. Both are operational frameworks that gain precision by restricting what questions can be asked.

Exercises

1. State precisely the criteria under which the Heisenberg cut may be placed in a Copenhagen analysis. What formal requirement must hold for the two sides of the cut to be "equivalent"?
2. Argue why decoherence makes the Heisenberg cut a defensible approximation but does not eliminate its arbitrariness.
3. For the Stern–Gerlach experiment, construct the joint state of atom + magnet before the detector screen measures the position. Verify that tracing out the magnet gives the same outcome probabilities as treating the magnet as a classical measuring device.
4. Explain why Copenhagen is formally incomplete (in the sense of Node 6.1) despite being empirically adequate.
5. In what sense is Copenhagen "realist" about the classical domain but not about the quantum domain? Is this combination internally consistent?
6. Compare the Copenhagen treatment of the measurement problem with its treatment of Bell nonlocality (Module 10.1). Is Copenhagen's refusal to assign pre-measurement values consistent with Bell's theorem?

7. Argue, structurally, why Copenhagen's operational minimalism has kept it the default interpretation of working physicists even as foundational work has moved beyond it.

Many-Worlds Interpretation

Position in Knowledge Graph

System: Quantum Foundations **Structural Domain:** Interpretations **Node:** 6.3

Many-worlds — originally due to Hugh Everett (1957) and developed by DeWitt, Wheeler, Deutsch, Wallace, Saunders, and others — is the interpretation that resolves the measurement problem by *dropping the projection postulate entirely.* Only the Schrödinger equation governs the evolution of the universe's total wavefunction; there is no collapse, no cut, no classical domain, and no preferred basis except the one dynamically selected by decoherence. What is experienced as measurement is the branching of the universal wavefunction into mutually decoherent sectors, each containing a definite outcome and a copy of the observer who saw it.

Many-worlds is the *minimal* interpretation in terms of postulates (it discards one) and the *maximal* interpretation in terms of ontology (everything in the wavefunction is real). It is the interpretation most directly coupled to the material of Module 5: decoherence and einselection are essential to its account of experience.

Formal Statement

The only postulate

The universe is described by a **universal wavefunction** $|\Psi\rangle$ on a Hilbert space that includes all physical systems, observers, apparatuses, and environment. This state evolves *only* under the Schrödinger equation:

$$i\hbar\frac{d}{dt}|\Psi\rangle = H|\Psi\rangle.$$

There is no projection postulate. There is no collapse. Measurement is just a unitary interaction between a system and an apparatus (and eventually an observer and environment), producing an entangled state in which the observer becomes correlated with the measurement outcome.

Branching

When a measurement is performed, the joint state of system + apparatus + observer + environment becomes, schematically,

$$|\Psi\rangle = \sum_i c_i\,|i\rangle_S \otimes |\text{apparatus shows } i\rangle_A \otimes |\text{observer sees } i\rangle_O \otimes |E_i\rangle_{\text{env}}.$$

Each term in this sum is a **branch** (or "world"). The environment states $|E_i\rangle$ are mutually orthogonal after decoherence (by the einselection argument of Node 5.5), so the branches have no practical way to interfere with each other. Within each branch, the observer has experienced a definite outcome i and will, from that moment on, describe the world in terms of that branch's history.

Everett's central claim: *each branch is equally real.* There is no "actual" outcome selected from among the branches — all of them occur, in different sectors of the universal wavefunction.

Probabilities

The coefficients $|c_i|^2$ are interpreted as probabilities — but the question of *why* they should be is non-trivial, because in a purely deterministic branching picture every outcome occurs. Several approaches to deriving the Born rule from the many-worlds framework have been developed:

- **Deutsch–Wallace decision-theoretic derivation** (2000–2012). A rational agent operating within the many-worlds framework can be shown, under plausible decision-theoretic axioms, to assign subjective

probabilities to branches equal to $|c_i|^2$. This is the formal analogue of the classical Cox/Savage derivations of Bayesian probability from rationality axioms.
- **Self-locating uncertainty** (Vaidman, Sebens–Carroll 2018). Immediately after branching but before the observer learns the outcome, they are uncertain which branch they are in. The correct probability to assign to being in a particular branch is derived from the structure of the wavefunction and turns out to equal $|c_i|^2$.
- **Envariance** (Zurek 2005). The Born rule follows from an invariance property of entangled states under local operations on one side balanced by compensating operations on the other.

These derivations are not universally accepted — critics argue that each presupposes in some form what it sets out to prove. The *status* of the Born rule in many-worlds is therefore a live area of foundational debate. What is not in dispute is that many-worlds' empirical predictions, if the Born rule is taken as an auxiliary postulate, match standard quantum mechanics exactly.

Intuition

The simplest reading: *take the Schrödinger equation seriously.* If a measurement is a physical process — and it is, since apparatuses are made of atoms — then it should be describable by unitary quantum mechanics all the way through, with no special exception for "measurement." But doing so produces entangled states of system, apparatus, and observer. Where does the definite outcome come from?

Everett's answer: nowhere. The entangled state *is* the situation, and each term in it is one branch of the universal wavefunction. The observer in branch 1 sees outcome 1; the observer in branch 2 sees outcome 2. Both observers are copies of the original observer before the measurement. Neither has any experiential access to the other branch — the branches have decohered, so no interference is possible.

From the inside of a branch, the experience of measurement is exactly what Copenhagen describes: an apparent random selection of one outcome from a distribution. The many-worlds account is that *each* branch looks locally like the Copenhagen account, but the global picture is unitary, branching, and deterministic.

The cost is an explosion of ontology. Every quantum event creates branches, and the total number of branches grows without bound. Many-worlds advocates reply that this is no more ontologically extravagant than committing to a continuous field theory in classical mechanics: in both cases, the theory is described by a single mathematical object, and whatever exists is whatever that object contains. The universe's "cost" is the dimension of its Hilbert space, not the number of terms in any particular decomposition.

The advantage is structural parsimony. Many-worlds has *one* dynamical rule (the Schrödinger equation) and no cuts, no collapse, no special role for observers. It is compatible with locality in a technical sense: there is no faster-than-light signalling, because branching is a local phenomenon that spreads at the speed of light. And it is compatible with determinism: the universal wavefunction evolves deterministically under the Schrödinger equation, even though each observer experiences apparent randomness.

Structural Role

Many-worlds is the interpretation of choice for certain classes of structural argument:

- **Dropping a postulate.** It is the *minimalist* response to the measurement problem in the specific sense that it removes one of the two rules of Node 6.1. This appeals to people who believe that physics should have the smallest possible set of fundamental laws.
- **Compatibility with quantum cosmology.** If the universe as a whole has a wavefunction (as in Wheeler–DeWitt quantum cosmology), there is no external observer to perform a measurement. Copenhagen becomes unusable; many-worlds provides a coherent framework in which the universal wavefunction just evolves.

- **Pairing with decoherence.** Many-worlds leans heavily on the machinery of Module 5. The preferred-basis problem ("why do we see position rather than position+momentum superpositions?") is answered by pointing to the pointer basis selected by the system–environment coupling. Decoherence is not a partial solution to the measurement problem in many-worlds; it is the *complete* account of why branches look definite from the inside.
- **Information-theoretic cousins.** Consistent-histories (Node 6.5), decoherent histories, and relational interpretations are close relatives of many-worlds in the sense that all drop the objective collapse and replace it with framework-dependent descriptions.
- **Controversies.** The Born-rule derivation remains contested. Critics include Price, Kent, Adlam, and others who argue that probabilities make sense only if something is uncertain, and in a fully deterministic many-worlds picture there is nothing to be uncertain about. Defenders respond via decision theory, self-locating uncertainty, or simply treating the Born rule as an auxiliary postulate.

Mathematical Development

The universal wavefunction

Let $\mathcal{H}_U$ be the Hilbert space of the universe, schematically $\mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_A \otimes \mathcal{H}_O \otimes \mathcal{H}_{\text{env}}$ (system, apparatus, observer, environment). The universal state $|\Psi\rangle \in \mathcal{H}_U$ evolves under H_U by the Schrödinger equation. All quantum predictions are to be computed from $|\Psi\rangle$ by taking inner products and tracing out unobserved degrees of freedom.

Everett's consistency claim. The empirical content of quantum mechanics — the probabilities of measurement outcomes — can be recovered from $|\Psi\rangle$ alone, without invoking any collapse. The recovery depends on:

1. The branches being decoherent (so no cross-branch interference is observable).
2. The Born-rule weighting of branches (which must be justified from the structure of $|\Psi\rangle$).

Branching as entanglement

Before a measurement, the state is $|\Psi_0\rangle = (\sum_i c_i|i\rangle_S) \otimes |0\rangle_A \otimes |0\rangle_O \otimes |E_0\rangle$. After the measurement interaction (a unitary that couples system to apparatus), the apparatus has become correlated with the system:

$$|\Psi_1\rangle = \sum_i c_i|i\rangle_S \otimes |a_i\rangle_A \otimes |0\rangle_O \otimes |E_0\rangle.$$

After the observer reads the apparatus, the observer is correlated as well:

$$|\Psi_2\rangle = \sum_i c_i|i\rangle_S \otimes |a_i\rangle_A \otimes |o_i\rangle_O \otimes |E_i\rangle,$$

where the environment states $|E_i\rangle$ are the einselected records of the observer's brain state interacting with its surroundings. Decoherence drives $\langle E_i|E_j\rangle \to 0$ on timescales of 10^{-13} seconds or faster.

Once the environment states are orthogonal, the reduced density matrix on the system + apparatus + observer is diagonal:

$$\rho_{SAO} = \sum_i |c_i|^2 |i\rangle\langle i|_S \otimes |a_i\rangle\langle a_i|_A \otimes |o_i\rangle\langle o_i|_O.$$

From the inside of any branch, the observer sees a classical mixture of outcomes with probabilities $|c_i|^2$ — exactly the Copenhagen prediction. No collapse was needed; the apparent randomness comes from the observer being in one branch and not having access to the others.

Preferred basis

The "preferred basis problem" is: why do branches decompose in the $\{|i\rangle\}$ basis rather than any other? The mathematical answer is decoherence + einselection (Node 5.5): the environment's coupling to the joint system determines the pointer basis, and that is the basis in which branching is well-defined. A basis that is not einselected would have its cross-terms remain coherent, making the "branches" interfere and rendering the description nonsense.

This is why many-worlds is structurally dependent on Module 5. Without decoherence, there would be no unique way to decompose the universal wavefunction into branches, and the many-worlds story would be ambiguous. With decoherence, the branches are the einselected components, and the branching structure is well-defined up to the limits of decoherence precision.

Deutsch–Wallace decision-theoretic derivation of Born rule

The Deutsch–Wallace program takes rationality axioms from classical decision theory (transitivity of preferences, continuity, Savage's sure-thing principle) and adds quantum-specific axioms (equivalence, additivity over branches, etc.). Under these axioms, a rational agent acting within the many-worlds framework must treat the branch amplitudes $|c_i|^2$ as probabilities. The derivation is technical and the axioms are contested, but the structural point is that the Born rule emerges from the combination of unitary dynamics and rational betting behavior — it is not an independent postulate.

Wallace's 2012 book *The Emergent Multiverse* gives the most developed form of this argument.

Self-locating uncertainty

The Sebens–Carroll approach uses the observation that, after the measurement interaction but before the observer learns the outcome, the observer is uncertain about which branch they are in — self-locating uncertainty. The natural probability assignment for being in branch i is then derived from symmetries of the wavefunction and, under plausible assumptions, equals $|c_i|^2$. The derivation is cleaner than Deutsch–Wallace's but still assumes something about how to count observers across branches.

Worked Example

Schrödinger's cat in many-worlds

Consider the full Schrödinger's cat setup: a radioactive atom in a state $(|\text{undecayed}\rangle + |\text{decayed}\rangle)/\sqrt{2}$, coupled to a Geiger counter, coupled to a vial of poison, coupled to a cat, coupled to Wigner, coupled to the environment. The joint unitary evolution takes the initial product state into

$$|\Psi\rangle = \tfrac{1}{\sqrt{2}}\big(|\text{undec}\rangle|G_0\rangle|V\rangle|\text{alive}\rangle|W_0\rangle|E_0\rangle + |\text{dec}\rangle|G_1\rangle|V_{\text{broken}}\rangle|\text{dead}\rangle|W_1\rangle|E_1\rangle\big),$$

where all the degrees of freedom have become entangled with the atom's decay status.

Copenhagen says: the cat is in a superposition until someone opens the box, at which point collapse happens.

Many-worlds says: there is no superposition-versus-collapse distinction. The universal wavefunction is always $|\Psi\rangle$. It contains two branches: one in which the atom did not decay and the cat is alive, and one in which it did and the cat is dead. Both branches are equally real. Before Wigner opens the box, he is in a factored state $|W_0\rangle$ not entangled with the atom; after he opens it, he becomes entangled:

$$|\Psi\rangle \to \tfrac{1}{\sqrt{2}}\big(|\text{alive}\rangle|W_{\text{sees alive}}\rangle + |\text{dead}\rangle|W_{\text{sees dead}}\rangle\big).$$

The "Wigner sees alive" copy of Wigner experiences a relieved observation; the "Wigner sees dead" copy experiences a sad one. Both exist. Each is only aware of his own branch.

Is this coherent? From the inside of each branch, the statistics match Copenhagen: subjective probability 1/2 for each outcome. From the outside (if there were some meta-observer), the universal wavefunction is a simple unitary evolution with no collapse. Many-worlds claims this is the only consistent picture.

The Wigner's-friend scenario

In the Frauchiger–Renner extension, one has Wigner measuring his friend who has measured a spin. In Copenhagen, there are delicate questions about whether the friend's "observation" counts as a collapse. In many-worlds, the question dissolves: after all the measurements, the universal wavefunction contains four branches corresponding to the four joint outcomes, all deterministic, all existing in parallel. The friend in each branch has a consistent story; the branches do not interfere.

The Frauchiger–Renner paradox, which claims to derive a contradiction from single-world assumptions plus quantum mechanics plus reasoning across observers, is consistent in many-worlds because many-worlds does not assume single-world outcomes.

Branching and the expansion of the universe

In many-worlds cosmology, the universal wavefunction branches constantly and everywhere. Each quantum event, from a cosmic ray decay to the spontaneous emission of a photon by a distant atom, creates branches. The total number of branches grows astronomically, but this is not a physical cost — it is just the dimension of Hilbert space being used. From an information-theoretic standpoint, the wavefunction has bounded Kolmogorov complexity (as a solution to a differential equation), even if the number of branches in any decomposition is large.

Cross-Links

- **Module 2.4 (Unitary evolution):** The only rule in many-worlds. Everything else follows.
- **Module 5.5 (Pointer basis / einselection):** Provides the preferred basis for branching. Many-worlds is structurally dependent on decoherence.
- **Module 6.5 (QBism / consistent histories):** Related agent-relative and framework-relative interpretations that also drop objective collapse.
- **Module 10 (Foundations):** The PBR theorem (10.3) constrains ψ-epistemic interpretations. Many-worlds is strongly ψ-ontic — the wavefunction is the only fundamental entity — and so survives PBR trivially.
- **Statistics:** Self-locating uncertainty and decision-theoretic derivations have direct analogues in Bayesian probability theory.

Structural Compression

Invariant: Only unitary evolution is fundamental. Measurement is a unitary interaction producing branches of the universal wavefunction; each branch is equally real; apparent randomness comes from being in one branch and not the others.

Mechanism: Schrödinger equation applied to the universal wavefunction, plus decoherence (Module 5) to select the preferred basis and make branches effectively independent.

Cross-System Echo: Bayesian self-locating uncertainty, statistical mechanics ensembles (imagined as many actual copies rather than one probabilistic system), and parallel computation all share the structural pattern "apparent selection arises from restriction to one sector of a larger space that contains all possibilities."

Exercises

1. Write out the universal wavefunction for Wigner's friend in full, tracking the entanglement structure at each step. Where does "collapse" appear in the many-worlds picture?
2. Explain why many-worlds does not require a preferred basis to be postulated — decoherence provides one dynamically. Under what conditions could the decoherence selection fail?
3. State the Deutsch–Wallace decision-theoretic argument for the Born rule in your own words. What is the key assumption being made about rational agency?
4. Argue why many-worlds is not refuted by the observation that we don't experience branching. (Hint: each branch is only aware of itself.)
5. Compare the many-worlds account of Schrödinger's cat with the Copenhagen account. Which of the questions "when did the cat become definite?" and "which cat am I?" is well-posed in each framework?
6. Critics argue that many-worlds' ontology is extravagant. Argue for or against this claim on structural grounds (e.g., Hilbert-space dimension, Kolmogorov complexity of the wavefunction).
7. Explain, structurally, why many-worlds requires Module 5 (decoherence) but not Module 6.5 (hidden variables). What is the relationship between the two?

Bohmian Mechanics

Position in Knowledge Graph

System: Quantum Foundations **Structural Domain:** Interpretations **Node:** 6.4

Bohmian mechanics — due to Louis de Broglie (1927) in its pilot-wave form and rediscovered and developed by David Bohm (1952), with later refinement by Bell, Dürr, Goldstein, Zanghì, and others — is the canonical *hidden-variable* interpretation of quantum mechanics. It restores determinism and particle trajectories by supplementing the wavefunction with a second physical ingredient: actual particle positions. The wavefunction evolves under the Schrödinger equation exactly as in standard quantum mechanics and plays the role of a guiding field. Particles follow definite trajectories determined by the wavefunction through a first-order *guidance equation*. All quantum predictions are recovered, provided one assumes that particle positions are distributed according to $|\psi|^2$ — the **quantum equilibrium hypothesis**.

Bohmian mechanics is the interpretation one reaches by insisting that physics describes an observer-independent reality of point particles moving in space, at the cost of accepting that the dynamics is explicitly nonlocal.

Formal Statement

The two dynamical objects

A Bohmian universe is specified by:

1. A **wavefunction** $\psi(x_1, \ldots, x_N, t)$ on configuration space, evolving under the usual Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \left(-\sum_{k=1}^{N} \frac{\hbar^2}{2m_k} \nabla_k^2 + V(x_1, \ldots, x_N) \right) \psi.$$

2. **Actual positions** $Q_1(t), \ldots, Q_N(t)$ of N particles, evolving under the **guidance equation**:

$$\frac{dQ_k}{dt} = \frac{\hbar}{m_k} \operatorname{Im} \left. \frac{\nabla_k \psi}{\psi} \right|_{x_1=Q_1,\ldots,x_N=Q_N}.$$

The wavefunction guides the particles; the particles do not back-react on the wavefunction. The theory is first-order in time (like a flow, not like Newtonian mechanics), and it is fully deterministic: given ψ and the initial positions $Q_k(0)$, the future evolution of everything is fixed.

Quantum equilibrium hypothesis

The statistical content of the theory comes from a single assumption about initial conditions: the positions of particles in any subsystem are distributed according to $|\psi|^2$ at some time. The continuity equation

$$\frac{\partial |\psi|^2}{\partial t} + \sum_k \nabla_k \cdot \left(|\psi|^2 \, v_k \right) = 0, \qquad v_k = \frac{\hbar}{m_k} \operatorname{Im} \frac{\nabla_k \psi}{\psi},$$

ensures that if $|\psi|^2$ holds initially, it holds for all time. This is called **equivariance**: the Bohmian flow carries the $|\psi|^2$ distribution into itself. The quantum equilibrium hypothesis is then the statement that our universe is in such an equilibrium.

Under quantum equilibrium, all empirical predictions of Bohmian mechanics agree exactly with those of standard quantum mechanics, because measurement outcomes — being reducible to final particle positions (pointer readings) — are distributed according to $|\psi|^2$. Non-equilibrium Bohmian mechanics, in which particle distributions differ from $|\psi|^2$, is a logically consistent possibility but is not observed (Valentini has argued it could leave cosmological signatures).

No separate measurement postulate

Bohmian mechanics has *no projection postulate.* Measurement is simply an interaction between a system and an apparatus in which the apparatus's pointer (a collection of Bohmian particles) ends up at one of several spatial locations. The Born rule is not postulated — it is *derived* from the guidance equation plus quantum equilibrium.

Intuition

The animating image is simple. Particles are real objects with definite locations at all times, moving through space along smooth trajectories. The wavefunction is a real field on configuration space that tells each particle how to move. A measurement outcome is just the final location of a pointer particle; there is nothing mysterious about it.

This is the interpretation a seventeenth-century natural philosopher would have wanted. It restores the classical picture almost in full: there are particles, they move, the trajectories exist, determinism holds. The only non-classical element is the nature of the guiding field, which lives on the $3N$-dimensional configuration space rather than on ordinary three-dimensional space and therefore couples distant particles instantaneously.

This instantaneous coupling is the price. Bell's theorem (Node 10.1) proves that any theory assigning definite values to measurement outcomes and reproducing quantum statistics must be nonlocal. Bohmian mechanics does not evade this — it *embraces* it. The guidance equation for particle k depends on the positions of all other particles simultaneously, so a measurement on one half of an entangled pair instantaneously alters the guidance of the other half. There is no signaling (the $|\psi|^2$ distribution prevents any observable faster-than-light effects), but the underlying dynamics is explicitly nonlocal in a way that sits uneasily with special relativity.

The second cost is mathematical elegance. The Schrödinger equation plus guidance equation is not the most economical description — it has two dynamical entities where standard quantum mechanics has one. And the quantum equilibrium hypothesis must be added as an independent postulate (though it has a statistical-mechanics-style justification in terms of typicality arguments).

The advantage is a clean ontology. Bohmian mechanics says exactly what exists (particles and wavefunction), exactly where it is (positions in configuration space), and exactly how it evolves (Schrödinger + guidance). There is no measurement problem because measurement is just a particular kind of interaction. There is no preferred-basis problem because the preferred basis is position, by fiat. There is no observer problem because observers are collections of Bohmian particles like any other.

Structural Role

Bohmian mechanics plays several distinctive structural roles in the foundations literature:

- **Existence proof for hidden variables.** Before Bohm, it was widely believed (following von Neumann's flawed 1932 "proof") that hidden-variable completions of quantum mechanics were impossible. Bohmian mechanics refuted this by exhibiting one. The von Neumann argument was shown by Bell to rest on an unjustified assumption (linearity of hidden-variable value assignments over non-commuting observables).
- **Counterexample to naïve locality claims.** Bohmian mechanics makes it concrete that nonlocal hidden-variable theories can reproduce quantum statistics. Bell's theorem was developed in part by thinking about what Bohmian nonlocality actually requires, and the Bell inequalities are tight: any theory reproducing quantum statistics and assigning definite outcomes must have this much nonlocality.
- **Position as the fundamental observable.** Bohmian mechanics takes position as the primitive ontology and derives the other observables' behavior from measurement dynamics. This makes it contextual in Bell's sense (Node 10.2): the "value" of, say, spin is not a property of the particle but of the joint system of particle plus measuring apparatus. This squares Bohmian mechanics with Kochen–Specker contextuality by accepting it straightforwardly.

- **ψ-ontic commitment.** The wavefunction in Bohmian mechanics is a real physical field, not an epistemic tool. This makes Bohmian mechanics strongly ψ-ontic, and it passes the PBR theorem (Node 10.3) trivially.
- **Contrast class to many-worlds.** Both Bohmian mechanics and many-worlds drop the projection postulate. Many-worlds keeps only the wavefunction; Bohmian mechanics keeps the wavefunction *and* adds particles. Defenders of many-worlds argue that Bohmian particles are redundant (the wavefunction is doing all the work); defenders of Bohmian mechanics argue that the particles are the only thing preventing the wavefunction from being a sum of unrealized possibilities.

Mathematical Development

Deriving the guidance equation

Write the wavefunction in polar form $\psi = Re^{iS/\hbar}$ with R, S real. Substituting into the Schrödinger equation and separating real and imaginary parts yields two equations:

$$\frac{\partial R^2}{\partial t} + \sum_k \nabla_k \cdot \left(R^2 \frac{\nabla_k S}{m_k} \right) = 0,$$

$$\frac{\partial S}{\partial t} + \sum_k \frac{(\nabla_k S)^2}{2m_k} + V + Q = 0, \qquad Q = -\sum_k \frac{\hbar^2}{2m_k} \frac{\nabla_k^2 R}{R}.$$

The first equation is a continuity equation for the density $R^2 = |\psi|^2$, with velocity field $v_k = \nabla_k S / m_k$. The second is a Hamilton–Jacobi equation for S supplemented by an additional term Q, the **quantum potential**, which is the only place $\hbar$ appears. Bohm's original 1952 formulation was cast in these terms: particles follow Newtonian-like trajectories under the combined potential $V + Q$.

The modern formulation (Dürr–Goldstein–Zanghì) drops the quantum potential and simply reads off the guidance equation directly from the probability current:

$$j_k = \frac{\hbar}{m_k} \operatorname{Im}(\psi^* \nabla_k \psi) = |\psi|^2 \frac{\nabla_k S}{m_k} = |\psi|^2 v_k,$$

giving

$$v_k = \frac{j_k}{|\psi|^2} = \frac{\hbar}{m_k} \operatorname{Im} \frac{\nabla_k \psi}{\psi}.$$

This form makes no reference to S (which is only defined up to $2\pi\hbar$ at nodes of ψ) and generalizes cleanly to spinors and many-particle systems.

Equivariance

If the distribution of particle positions is $\rho(x, t)$, then ρ satisfies a continuity equation:

$$\frac{\partial \rho}{\partial t} + \sum_k \nabla_k \cdot (\rho v_k) = 0.$$

Since $|\psi|^2$ also satisfies this equation with the same velocity field (from the Schrödinger equation), any distribution equal to $|\psi|^2$ at one time remains equal to $|\psi|^2$ at all later times. This is *equivariance*, and it is the technical basis for the quantum equilibrium hypothesis being consistent.

Quantum equilibrium and typicality

The question "why are particles distributed according to $|\psi|^2$?" has been addressed by Dürr–Goldstein–Zanghì through a **typicality argument**, analogous to Boltzmann's justification of the uniform distribution on phase space in classical statistical mechanics. The argument is: among the measure-$|\Psi|^2$ set of possible initial particle configurations of the universe, the vast majority of initial conditions lead to subsystem distributions that are empirically indistinguishable from $|\psi|^2$. So quantum equilibrium is *typical* in a precise measure-theoretic sense, even if it is not inevitable.

Nonlocality in entangled systems

For a two-particle entangled state $\psi(x_1, x_2)$, the guidance equation for particle 1 depends on x_2:

$$\dot{Q}_1 = \frac{\hbar}{m_1} \operatorname{Im} \left. \frac{\nabla_1 \psi(x_1, x_2)}{\psi(x_1, x_2)} \right|_{x_1 = Q_1, x_2 = Q_2}.$$

If ψ does not factorize, the velocity of particle 1 depends on where particle 2 is *right now.* Measuring particle 2 (which in Bohmian mechanics means causing its position to be correlated with a macroscopic pointer) alters the effective wavefunction that guides particle 1, and this alteration propagates instantaneously.

This is the explicit nonlocality Bell's theorem requires of any definite-outcome theory. The theory is compatible with no-signaling because the $|\psi|^2$ distribution over hidden initial conditions averages out the nonlocal influences at the statistical level — any experimenter making only macroscopic measurements cannot use the nonlocal dynamics to send information.

Measurement as entanglement plus position readout

Consider a spin measurement in the Bohmian picture. The spin-1/2 particle has a wavefunction $\psi(x) = \alpha\phi_+(x) + \beta\phi_-(x)$ where $\phi_\pm$ are spin eigenstates with spatial wavepackets. A Stern–Gerlach magnet couples spin to position, producing after evolution a bipartite wavefunction in which spin-up packets are at position $+z$ and spin-down packets are at position $-z$:

$$\psi(x, t) = \alpha\phi_+(x - z_0\hat{z}, t) + \beta\phi_-(x + z_0\hat{z}, t).$$

The actual Bohmian particle ends up in one of the two packets — whichever one contains its initial position. Which one it is depends deterministically on the initial conditions, but because those conditions are distributed according to $|\psi|^2$, the observed statistics are $|\alpha|^2$ and $|\beta|^2$. The spin "value" is not a property of the particle; it is a feature of the apparatus–particle interaction that happens to correlate cleanly with the final position.

Worked Example

Double-slit with Bohmian trajectories

A particle impinges on a screen with two slits. The wavefunction after the slits is the superposition $\psi = \psi_1 + \psi_2$ of two outgoing wavepackets. Downstream, the two packets overlap and interfere, producing the standard $\cos^2$ fringe pattern in $|\psi|^2$.

In Bohmian mechanics, the particle has a definite position at all times. It goes through one slit — slit 1 if its initial position is in the left half of the configuration space, slit 2 otherwise. After the slits, the particle's trajectory is guided by the total ψ, and in the overlap region the trajectories become strongly curved, bunching toward the bright fringes and avoiding the dark fringes.

Numerical computations of these trajectories (originally by Philippidis, Dewdney, and Hiley in 1979) show a distinctive "no-crossing" property: Bohmian trajectories never cross each other in configuration space. This means that in the symmetric double-slit, a particle going through the left slit always ends up on the left half of the screen, and vice versa. (Contrast with the Feynman picture, where paths cross freely.)

The statistical distribution of impact positions is $|\psi|^2$, reproducing the interference pattern exactly. An individual particle's path is a definite curve, but which curve depends on the initial position, and the ensemble of curves over the $|\psi|^2$ distribution produces interference.

Crucially, *which slit the particle went through* is not observable without disturbing the wavefunction. Any detector at one slit destroys the coherence of the two packets and thus destroys the interference. Bohmian mechanics is consistent with the standard experimental phenomenology: in the interference case, the question "which slit did it go through?" has an *answer* according to the theory (one can in principle compute it from initial conditions), but no *measurement* can reveal that answer without erasing the interference. This is the sense in which Bohmian hidden variables are "hidden."

EPR in Bohmian mechanics

Consider an entangled singlet pair $|\psi\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$. Alice measures her particle's spin along some axis; Bob measures his along another.

In Bohmian mechanics, both particles have definite position trajectories throughout. When Alice performs her measurement, her apparatus couples to her particle's spin-position correlation, and her particle ends up in one of the spin-up or spin-down packets. The *wavefunction* is still an entangled two-particle wavefunction, so the "effective wavefunction" guiding Bob's particle is instantaneously updated: Bob's particle is now guided by the conditional wavefunction corresponding to Alice's outcome.

This update is nonlocal in the strict sense — it happens at Bob's location at the same time as Alice's measurement, regardless of spatial separation. But the statistics of Bob's subsequent measurement remain 1/2, 1/2 if he hasn't yet been informed of Alice's outcome, so no signal has been sent. The nonlocality is "under the hood": visible in the equations of motion of hidden variables but invisible at the level of observed statistics.

Bell's theorem is what *forces* this nonlocality: if Bohmian mechanics were local, it could not reproduce the violations of CHSH observed in real EPR experiments. The theorem guarantees that any deterministic hidden-variable theory with definite outcomes must have this feature. Bohmian mechanics has it explicitly and unapologetically.

Hydrogen ground state

The hydrogen ground-state wavefunction $\psi = (\pi a_0^3)^{-1/2} e^{-r/a_0}$ is real. Since the guidance equation is proportional to $\mathrm{Im}(\nabla\psi/\psi)$, and the wavefunction is real, the velocity field vanishes identically. Therefore the Bohmian electron in the ground state is *at rest*: it sits at some definite position in the cloud and does not move at all.

This is unintuitive from a classical standpoint — a stationary electron in a hydrogen atom? — but consistent. The orbital picture of an electron orbiting the nucleus does not correspond to Bohmian reality. The "motion" in the classical picture is really the spread of the wavefunction; the Bohmian particle itself is stationary in s-states. Kinetic energy appears through the quantum potential Q, not through actual motion. This is one of the places where Bohmian mechanics' ontology diverges most strikingly from classical intuition.

Cross-Links

- **Module 6.1 (Measurement problem):** Bohmian mechanics resolves the measurement problem by eliminating the collapse postulate and treating measurement as an interaction that correlates the apparatus position with the system state.

- **Module 6.3 (Many-worlds):** Both drop the projection postulate. Many-worlds keeps only the wavefunction; Bohmian mechanics adds particles. Structurally, Bohmian mechanics can be seen as "many-worlds with a marker pointing to the branch we're in."
- **Module 10.1 (Bell's theorem):** Forces Bohmian mechanics to be nonlocal. The nonlocality is visible in the guidance equation for entangled states.
- **Module 10.2 (Kochen–Specker):** Bohmian mechanics is contextual: the "value" of a spin measurement depends on the apparatus, not just on the particle. This is consistent with KS, which forbids non-contextual hidden variables.
- **Module 10.3 (PBR theorem):** Bohmian mechanics is ψ-ontic (the wavefunction is real) and so survives PBR.
- **Classical mechanics:** The guidance equation is first-order in time, like a classical flow, and the quantum potential plays a role analogous to a Hamilton–Jacobi potential.
- **Statistics:** Quantum equilibrium is analogous to thermodynamic equilibrium in statistical mechanics — a typical distribution that dominates observations even though non-equilibrium initial conditions are logically possible.

Structural Compression

Invariant: Particles have definite positions at all times, guided by a wavefunction that evolves unitarily. Measurement reveals position; other "values" (spin, momentum) are contextual features of the particle–apparatus interaction. Quantum statistics are recovered because positions are distributed according to $|\psi|^2$.

Mechanism: Schrödinger equation for ψ, guidance equation for particle positions, quantum equilibrium hypothesis $\rho = |\psi|^2$ as the initial condition.

Cross-System Echo: Latent-variable models in statistics, hidden Markov state in Bayesian filtering, and stochastic particle methods in fluid dynamics all share the pattern "guiding field + tracer particle distributed by that field's density."

Exercises

1. Derive the guidance equation from the Schrödinger equation via the polar decomposition $\psi = Re^{iS/\hbar}$. Verify that the velocity field is $\nabla S/m$ and that $|\psi|^2$ satisfies a continuity equation with this velocity.
2. Show that the hydrogen ground-state wavefunction, being real, yields zero Bohmian velocity. Discuss the tension with the classical picture of an orbiting electron.
3. For a particle in a one-dimensional box in the n-th stationary state, compute the Bohmian trajectories. Verify that the particle is at rest, and explain how this is consistent with nonzero kinetic energy expectation values.
4. Construct the Bohmian trajectories for a Gaussian wavepacket in free space. Show that the trajectories spread linearly with time, matching the $|\psi|^2$ spread.
5. For an entangled singlet pair, write the guidance equation for each particle and show explicitly that it depends on the other particle's position. Use this to explain how Bohmian mechanics is nonlocal yet no-signaling.
6. Discuss the quantum equilibrium hypothesis. What would it mean for the universe to *not* be in quantum equilibrium, and what observable signatures would such a state have?
7. Argue, structurally, why Bohmian mechanics accepts the results of the Kochen–Specker theorem as a feature rather than a problem. What does contextuality mean in the Bohmian framework?

Consistent Histories and QBism

Position in Knowledge Graph

System: Quantum Foundations **Structural Domain:** Interpretations **Node:** 6.5

Two twentieth-century interpretive frameworks developed after Copenhagen, many-worlds, and Bohmian mechanics, each dissolving the measurement problem in a different way.

Consistent histories — due to Robert Griffiths (1984), Roland Omnès, and in a closely related form to Murray Gell-Mann and James Hartle — is a framework in which quantum mechanics is formulated as a theory about sequences of events (histories) rather than states at fixed times. Classical probabilities are assigned to histories, but only within self-consistent *frameworks*, and different frameworks may describe the same physical situation in mutually exclusive ways. There is no objective collapse; there is no single universal description; there is only the choice of framework plus decoherence.

QBism (Quantum Bayesianism) — due to Christopher Fuchs, Rüdiger Schack, and David Mermin — is a radically agent-centered reading in which the wavefunction is not a physical property of any system but a personal degree of belief held by an individual agent about their own future measurement outcomes. The Born rule is a normative constraint on rational beliefs, and "collapse" is just Bayesian updating when the agent learns a new fact. Measurement is the agent's personal experience.

Both approaches abandon the idea of an observer-independent objective wavefunction. Both reproduce all standard predictions. Both draw on Bayesian probability theory as an organizing principle. They differ in whether there is a shared objective world at all (CH says yes, subject to framework choice; QBism says the only thing "objective" is the formalism itself).

Formal Statement

Consistent histories

A **history** H_α is a sequence of projection operators $P^{(1)}_{\alpha_1}, P^{(2)}_{\alpha_2}, \ldots, P^{(n)}_{\alpha_n}$ at times $t_1 < t_2 < \cdots < t_n$, where each $P^{(k)}_{\alpha_k}$ projects onto some subspace of the Hilbert space at time t_k. Intuitively, H_α describes a possible sequence of events: "at t_1 the property α_1 held, then at t_2 the property α_2 held, and so on."

The **class operator** for a history is the time-ordered product

$$C_\alpha = P^{(n)}_{\alpha_n}(t_n)\, P^{(n-1)}_{\alpha_{n-1}}(t_{n-1}) \cdots P^{(1)}_{\alpha_1}(t_1),$$

in the Heisenberg picture (so each $P^{(k)}_{\alpha_k}(t_k) = U(t_k)^\dagger P^{(k)}_{\alpha_k} U(t_k)$ for unitary U).

The **decoherence functional** on a set of histories is

$$D(\alpha, \beta) = \mathrm{Tr}\left(C_\alpha\, \rho_0\, C^\dagger_\beta\right),$$

where ρ_0 is the initial state. The set of histories is **consistent** (or **decoherent**) if

$$\mathrm{Re}\ D(\alpha, \beta) = 0 \quad \text{whenever } \alpha \neq \beta.$$

Equivalently, the diagonal of the decoherence functional gives real nonnegative probabilities $p(\alpha) = D(\alpha, \alpha)$ that sum to 1 and satisfy the classical sum rules. A stronger condition, *strong decoherence*, requires $D(\alpha, \beta) = 0$ for $\alpha \neq \beta$.

Framework-dependence. Two different sets of histories may each be consistent, yet make contradictory claims about what happened. Consistent histories accepts this: the "correct" description of a physical situation depends on the framework (choice of projector set at each time), and different frameworks are mutually incompatible but each internally valid.

QBism

In QBism, a quantum state $|\psi\rangle$ (or density operator ρ) is not a property of any system. It is a **catalogue of personal probabilities** that an agent assigns to the possible outcomes of measurements they might perform. When the agent performs a measurement and learns the result, they update $|\psi\rangle$ by Bayesian conditionalization on the outcome — this is "collapse," and it is not a physical process but a belief revision.

The Born rule is a normative rule: given that an agent holds belief state ρ and will measure POVM $\{E_i\}$, their probabilities for outcomes must be $p(i) = \mathrm{Tr}(\rho E_i)$. Deviating from this rule would make the agent susceptible to a Dutch book — a set of bets guaranteed to lose.

SIC-POVMs (symmetric informationally complete POVMs) play a privileged role in QBism: a SIC-POVM in d-dimensional Hilbert space consists of d^2 rank-one projectors $\{|\phi_i\rangle\langle\phi_i|/d\}$ with $|\langle\phi_i|\phi_j\rangle|^2 = 1/(d+1)$ for $i \neq j$. Any state ρ can be reconstructed from the probabilities it assigns to a SIC measurement, and the Born rule can be rewritten as an "urgleichung" that looks like a deformation of classical total probability. QBists read this as evidence that quantum mechanics *is* Bayesian probability theory for agents in a fundamentally stochastic world, with the deformation encoding the constraints of unitarity.

No objective collapse

Both frameworks reject objective collapse: CH because there is no single objective wavefunction at all, only framework-relative histories; QBism because the wavefunction is personal, so "collapse" is just the agent updating their own beliefs. In both cases, the measurement problem — how to reconcile unitary evolution with definite outcomes — dissolves, because one of the two rules is reinterpreted as something other than a physical dynamical process.

Intuition

Consistent histories. Imagine watching a movie of a quantum system. At each time step, you have a choice of coarse-graining: which properties to "ask about." A framework is a consistent choice of questions at each time, such that the answers can be assigned joint classical probabilities. The key insight is that there is no single privileged framework — you can ask "was the particle in the left half?" at t_1, or you can ask "what was its momentum?" at t_1, but not both in the same framework, because the two questions correspond to non-commuting projectors and do not satisfy the consistency condition. Consistent histories tells you: pick a framework, compute your probabilities, do not mix frameworks. This is a formalization of Copenhagen's contextual "you have to specify your experimental setup" rule, extended to multiple times and framed as a choice of classical sub-theory within quantum mechanics.

QBism. Imagine you are an agent with a catalogue of beliefs about what the universe will do next. Some of your beliefs are "classical" (the coin will land heads with probability 1/2), and some are "quantum" (this photon will be detected at the upper detector with probability $|\alpha|^2$). The quantum beliefs are just beliefs; they live in your head. The universe is not in a quantum state — only *your description of it* is. When you measure, you learn something new, and you update your beliefs. The next time someone asks you to predict something, you use your updated beliefs. There is no physical collapse because there was never a physical wavefunction. The measurement problem dissolves by relocating the wavefunction from the world to the agent's belief state.

Both interpretations take the Bayesian analogy seriously. In frequentist statistics, probabilities are objective features of the world; in Bayesian statistics, they are degrees of belief. The tension between "collapse is a real process" and "collapse is an update rule" mirrors the frequentist-versus-Bayesian tension in statistics, and both CH and QBism are Bayesian in spirit.

The disagreement between CH and QBism is over *whose* beliefs and *how shared* they are. Consistent histories is "objective Bayesian" in the Jaynes sense — the framework is a choice, but once you pick it, the probabilities are determined by the quantum formalism and are the same for anyone using that framework. QBism is "subjective Bayesian" — each agent has their own personal state, and there is no requirement that agents agree.

Structural Role

- **Dissolving the measurement problem through framework relativity.** Both CH and QBism deny that there is a single objective physical state that must either collapse or not. CH says: there are multiple consistent descriptions, each internally classical; the "measurement problem" arises only if you try to impose a single description. QBism says: there is one description per agent, and "measurement" is that agent's own experience.
- **Formalizing Copenhagen.** Consistent histories can be read as a mathematically precise version of Copenhagen's complementarity: Bohr's choice of experimental setup becomes the framework choice, and the Heisenberg cut becomes the coarse-graining at each time. QBism can be read as the radicalization of Copenhagen's epistemic reading: the wavefunction really is only knowledge, and that knowledge belongs to a specific agent.
- **Bayesian reconstruction programs.** QBism is closely tied to attempts to *derive* the Born rule from decision theory and Bayesian rationality (Fuchs's work on SIC-POVMs, Appleby, and others). If successful, these would show that quantum mechanics is the unique consistent extension of Bayesian probability to a world with certain non-classical constraints.
- **Compatibility with special relativity.** Both CH and QBism are naturally local in the sense that there is no objective faster-than-light influence: CH because histories are framework-relative and the framework is a choice made by the analyst; QBism because updates are agent-local and there is no "collapse" propagating across space. This contrasts with Bohmian mechanics, which is explicitly nonlocal.
- **Criticisms.** Consistent histories is accused of proliferating frameworks (there are many consistent sets, and the formalism does not say which to use). QBism is accused of solipsism (if the wavefunction is personal, is the world real at all?) — a charge Fuchs and Mermin dispute by distinguishing between the wavefunction (agent-centric) and the world (which is real but not captured by the wavefunction).

Mathematical Development

The decoherence functional and consistency

Let ρ_0 be the initial density operator and C_α the class operator for history α. The probability assigned to history α is

$$p(\alpha) = \mathrm{Tr}(C_\alpha \rho_0 C_\alpha^\dagger).$$

For these probabilities to obey the classical sum rule — i.e., for the probability of the union of two disjoint histories to be the sum of their individual probabilities — the off-diagonal decoherence functional must vanish:

$$D(\alpha, \beta) = \mathrm{Tr}(C_\alpha \rho_0 C_\beta^\dagger) \approx 0, \qquad \alpha \neq \beta.$$

When this holds, the set is consistent and classical probability reasoning applies within it. Typically, decoherence (Module 5) provides the mechanism: if each projector at each time is coarse-grained over a pointer-basis partition of an environment, the decoherence functional suppresses off-diagonal elements exponentially in the environment size.

Framework examples

Consider a qubit in the initial state $\rho_0 = |+\rangle\langle+|$, evolved trivially (no dynamics), with two projector sets at a single time:

- Framework A: $\{P_0 = |0\rangle\langle 0|, P_1 = |1\rangle\langle 1|\}$. Probabilities: $p(0) = p(1) = 1/2$.
- Framework B: $\{P_+ = |+\rangle\langle+|, P_- = |-\rangle\langle-|\}$. Probabilities: $p(+) = 1, p(-) = 0$.

Both frameworks are consistent and give valid descriptions. They are mutually incompatible: framework A says "the bit is 0 with probability 1/2," framework B says "the bit is + with certainty." Consistent histories says: pick one. Do not try to assign joint probabilities across frameworks — that would correspond to non-commuting projectors and would violate consistency.

SIC-POVMs and the Born rule in QBism

A SIC-POVM in $\mathcal{H}_d$ is a set of d^2 rank-1 operators $E_i = \Pi_i/d$ with $\Pi_i = |\phi_i\rangle\langle\phi_i|$ and pairwise overlaps $|\langle\phi_i|\phi_j\rangle|^2 = 1/(d+1)$ for $i \neq j$. A state ρ can be reconstructed from its SIC probabilities $p_i = \mathrm{Tr}(\rho E_i)$ as

$$\rho = (d+1)\sum_i p_i \Pi_i - \mathbb{1}.$$

In QBism, these SIC probabilities are the fundamental objects: the agent's belief state is specified by giving $(p_1, \ldots, p_{d^2})$. The Born rule for an arbitrary measurement $\{M_j\}$ can then be rewritten as

$$q(j) = \sum_i \left[(d+1)p_i - \frac{1}{d}\right] r(j|i),$$

where $r(j|i) = \mathrm{Tr}(\Pi_i M_j)$ is the conditional probability of the outcome j given the "reference measurement" outcome i. This looks like classical total probability $q(j) = \sum_i p_i r(j|i)$ with a characteristic quantum correction. QBists argue this form shows that quantum mechanics is "Bayesian probability with a twist" — the twist being the specific deformation required by Hilbert space structure.

Dutch book argument for the Born rule

If an agent's betting quotients on quantum outcomes do not satisfy the Born rule, a clever opponent can construct a combination of bets that guarantees the agent loses money regardless of outcomes. This is the Dutch book argument, and in QBism it is the normative basis for the Born rule: a rational agent is one whose betting is not Dutch-bookable, and for quantum measurements the unique Dutch-book-free assignment is $p_i = \mathrm{Tr}(\rho E_i)$.

Worked Example

EPR in QBism

Consider an EPR singlet $|\psi\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$ shared between Alice and Bob. Alice measures her qubit in the Z basis and obtains outcome 0.

In Copenhagen, one would say Bob's qubit has "collapsed" to $|1\rangle$, and this collapse is nonlocal and puzzling (how does Bob's qubit know what Alice measured?).

In QBism: the wavefunction is Alice's personal belief state about her own future measurements. Before her measurement, she held the belief $|\psi\rangle$ about the joint system; this is a statement about her own betting odds for various joint outcomes, nothing more. When she measures and obtains 0, she updates her belief state. Her new belief about Bob's qubit is $|1\rangle$ — but this is *Alice's* belief, held in Alice's head, not a physical property of Bob's qubit.

Bob, from his perspective, holds his own belief state. He has not yet measured, so his belief about his qubit is still the reduced density matrix $I/2$ derived from the singlet. There is no moment at which Bob's qubit physically collapses, because there is no physical wavefunction. When Alice tells Bob her outcome, Bob updates his belief to match Alice's prediction, but this update is local — the information traveled classically from Alice to Bob.

The nonlocality of EPR, in QBism, is the nonlocality of correlations, not of causal influences. No physical thing happens at Bob's location at the moment of Alice's measurement. The only thing that happens is that Alice updates her personal belief state — and that belief state lives in Alice's head, not in the world. Bell's theorem is a constraint on the joint probability tables of the agents' beliefs, not on hidden variables or causal propagation.

Double-slit in consistent histories

Consider a particle passing through a double-slit apparatus with two possible paths. Three frameworks:

Framework A (which-path): projectors P_L, P_R onto the left/right slit at the intermediate time, plus final position at the screen. If a which-path detector is present, this framework is consistent (decoherence makes $D(L, R) \approx 0$ on the screen), and probabilities add classically: the interference pattern disappears.

Framework B (no which-path question): only project on the screen position at the final time. This framework is consistent (trivially, since it is a single-time measurement), and the probability distribution is the full interference pattern $|\psi_L + \psi_R|^2$.

Framework C (which-path and screen, no detector): tries to assign joint probabilities to "the particle went left" and "it hit position x on the screen." This framework is *not* consistent when there is no decoherence between slit paths, because $D(L, R) = \mathrm{Tr}(C_L \rho_0 C_R^\dagger)$ is generically nonzero due to interference. So one is forbidden from asking "which slit did it go through and where did it land?" in the absence of a which-path detector — not because of physical impossibility but because the joint question does not correspond to a consistent history.

This recovers the standard quantum story: you can talk about which slit (and destroy interference via a detector that decoheres the path) or about screen position with interference, but not both. Consistent histories formalizes this as a rule about which frameworks are allowed.

Cross-Links

- **Module 5.4 (Decoherence mechanisms):** The decoherence functional in consistent histories is driven by the same physical decoherence that selects pointer bases. Consistency is the formal analogue of "branches don't interfere."
- **Module 6.2 (Copenhagen):** Both CH and QBism can be read as mathematical refinements of Copenhagen. CH formalizes the framework-dependence; QBism formalizes the epistemic reading.
- **Module 6.3 (Many-worlds):** Consistent histories can be seen as "decoherent many-worlds without the ontological commitment to all branches being real." QBism goes in the opposite direction — there are no branches, only agent beliefs.
- **Module 10.1 (Bell's theorem):** QBism sidesteps Bell nonlocality by treating the wavefunction as personal; CH sidesteps it by treating frameworks as choices rather than objective facts. Both deny the premise of Bell's theorem that there is an observer-independent joint probability distribution.
- **Module 10.4 (SIC-POVMs):** QBism's mathematical core rests on SIC-POVM structures, which are conjectured to exist in every finite dimension.
- **Statistics — Bayesian inference:** QBism is explicit about its Bayesian character. The Born rule plays the role of a Dutch-book-free prior; measurement updates are Bayesian conditionalization.

Structural Compression

Invariant: There is no objective wavefunction collapse. Quantum mechanics is a framework for assigning (framework-relative or agent-relative) probabilities to events. The measurement problem dissolves because one of its premises — that there is a single objective wavefunction that must either evolve unitarily or collapse — is denied.

Mechanism: Consistent histories: decoherence functional $D(\alpha, \beta)$ vanishing off-diagonal within a chosen framework of projector sequences. QBism: the wavefunction is an agent's personal belief state, the Born rule is a Dutch-book-free constraint, and measurement updates are Bayesian conditionalization.

Cross-System Echo: Bayesian probability theory, in which probabilities are degrees of belief and updates are conditionalizations, is the direct structural cousin of QBism. Framework-relative probability assignments in consistent histories echo the structure of reference frames in relativity — different observers may use different coordinates, and no single one is privileged, but each is internally consistent.

Exercises

1. Write out the decoherence functional explicitly for a two-time history on a qubit with initial state $|+\rangle$, projectors P_0, P_1 at time t_1, and P_+, P_- at time t_2. Determine whether the set is consistent.
2. Show that the decoherence functional's diagonal entries give probabilities that sum to 1 whenever the projectors at each time form a complete set of orthogonal projectors.
3. Construct two consistent frameworks for a qubit that assign mutually contradictory probabilities to the same "event." Explain why this is not a contradiction in consistent histories.
4. For the SIC-POVM in $d = 2$, write out the four projectors explicitly (they are rank-1 projectors onto the vertices of a regular tetrahedron inscribed in the Bloch sphere). Verify the overlap condition.
5. Explain the Dutch book argument for the Born rule: why would an agent with non-Born-rule beliefs lose money to a clever opponent?
6. Describe the EPR experiment from a QBist perspective. In what sense is there no nonlocality, and what does Bell's theorem become in this framing?
7. Argue, structurally, why consistent histories and QBism can be seen as Bayesian refinements of Copenhagen. What does each take from the Copenhagen tradition, and what does each add?

Empirical Equivalence and Constraints

Position in Knowledge Graph

System: Quantum Foundations **Structural Domain:** Interpretations **Node:** 6.6

The four interpretations developed in Module 6 — Copenhagen (6.2), many-worlds (6.3), Bohmian mechanics (6.4), and consistent histories / QBism (6.5) — are *empirically equivalent* in the precise sense that each reproduces the Born-rule predictions of standard quantum mechanics exactly. No experiment within the scope of standard quantum mechanics can distinguish them. And yet they make radically different claims about what exists: one wavefunction or many, with or without particles, with or without collapse, with or without framework relativity.

This node closes Module 6 by making the empirical-equivalence situation structurally precise and by identifying the no-go theorems that *do* constrain the space of admissible interpretations. The full formal content of these theorems is developed in Module 10, but their structural role — as filters on interpretive space — belongs here.

The punchline: no interpretation of standard quantum mechanics has been ruled out by experiment, and current no-go theorems eliminate only certain broad structural classes of hidden-variable theories. Choice of interpretation is therefore metaphysical, not empirical, but the metaphysical options are constrained rather than unconstrained.

Formal Statement

Empirical equivalence

Definition. Two interpretations I_1 and I_2 of standard quantum mechanics are *empirically equivalent* if, for every experimental setup and every quantum-mechanical measurement, the probability distribution over outcomes predicted by I_1 coincides exactly with that predicted by I_2 — and with the standard Born-rule prediction.

All four interpretations of Module 6 (and many related ones: relational quantum mechanics, modal interpretations, transactional, etc.) satisfy this definition with respect to standard non-relativistic quantum mechanics. So do objective-collapse theories (GRW, Penrose) to within current experimental precision, though these make predictions that differ from standard QM and are thus only *approximately* empirically equivalent.

The no-go theorems

The following three theorems (formalized in Module 10) act as structural filters on the space of interpretations:

Bell's theorem (Node 10.1). Any hidden-variable theory reproducing quantum statistics and assigning definite values to all measurement outcomes must be nonlocal: there must be a physical influence between spacelike-separated measurement events, in the form of dependence of one measurement's outcome on the setting of a distant measurement's apparatus. Formally: no local hidden-variable theory can reproduce the CHSH violations observed in Bell experiments.

Kochen–Specker theorem (Node 10.2). Any hidden-variable theory in Hilbert space dimension $d \geq 3$ that assigns pre-existing values to all observables must be *contextual*: the value assigned to an observable may depend on which other (commuting) observables are measured along with it. Non-contextual hidden variables are impossible.

PBR theorem (Node 10.3) — Pusey, Barrett, Rudolph (2012). Under the preparation independence assumption, any hidden-variable theory in which the quantum state is ψ-epistemic (i.e., two distinct pure states can correspond to overlapping probability distributions over hidden variables) contradicts quantum predictions. Therefore, subject to the premises, the quantum state must be ψ-ontic: distinct pure states correspond to disjoint ontic states.

Auxiliary results:

- **Free Will Theorem** (Conway–Kochen 2006, 2009). If experimenters have free will to choose measurement settings, then particles must also have an analogous "free will" (outcomes not determined by past history on the light cone). A strengthening of Bell.
- **Frauchiger–Renner** (2018). A thought experiment showing that single-world assumptions, quantum mechanics applied to observers, and certain reasoning principles jointly lead to contradictions. Each interpretation handles this differently.

Filter structure

Each theorem eliminates a specific class of interpretations:

- Bell eliminates **local hidden-variable theories.**
- Kochen–Specker eliminates **non-contextual hidden-variable theories** (in $d \geq 3$).
- PBR eliminates a large class of **ψ-epistemic hidden-variable theories.**

What remains is the space of admissible interpretations: nonlocal or contextual hidden-variable theories (Bohmian), ψ-ontic theories without hidden variables (many-worlds), or theories that reject one of the premises of the no-go theorems (QBism rejects the "observer-independent joint distribution" premise, and in that sense evades rather than satisfies the theorems).

Intuition

The empirical-equivalence situation is sometimes described as a scandal: how can multiple mutually exclusive accounts of reality all make the same predictions? The structural answer is that standard quantum mechanics is, itself, under-committal about ontology. It specifies a formalism (Hilbert space + Born rule) and tells you how to use it to predict experiments. The *interpretations* are different ways of filling in a metaphysical story that the formalism alone leaves open. Since the formalism determines the predictions and the predictions are what experiments measure, adding extra metaphysics on top of the formalism cannot change experimental outcomes.

But "empirically equivalent" does not mean "equally good" or "equally plausible." Interpretations can be compared along axes other than prediction:

- **Ontological economy.** How much extra machinery beyond the formalism does the interpretation require? Bohmian mechanics adds particle positions; many-worlds subtracts the collapse rule and claims branches are real; QBism relocates the wavefunction to the agent's head.
- **Conceptual clarity.** Does the interpretation dissolve the measurement problem, or merely relocate it?
- **Compatibility with relativity.** Bohmian mechanics is explicitly nonlocal and fits uneasily with Lorentz invariance. Many-worlds and QBism are more naturally relativistic.
- **Compatibility with cosmology.** Interpretations requiring an external observer or classical apparatus (Copenhagen) struggle with cosmological contexts where there is no such external structure.
- **Derivation of the Born rule.** Some interpretations take Born as axiomatic; others attempt to derive it from decision theory (Deutsch–Wallace), self-location (Sebens–Carroll), or Dutch-book arguments (QBism).

The no-go theorems narrow this space. Before Bell (1964), it was possible to hope for a local classical picture behind quantum statistics; Bell proved this hope false. Before KS (1967), it was possible to hope for non-contextual hidden variables; KS proved this hope false in dimension ≥ 3. Before PBR (2012), it was possible to hope for a broad class of ψ-epistemic theories; PBR constrained them. Each theorem is a reduction in the space of viable stories.

What remains is: if you want classical hidden variables, you must accept nonlocality and contextuality (Bohmian mechanics does this). If you want to keep locality, you must give up single-world definite outcomes (many-worlds) or relocate the state to the agent (QBism). If you want to keep both locality and a single world, you must give up some other principle (e.g., counterfactual definiteness). There is no option that keeps everything.

Structural Role

- **Closing the module.** This node states clearly: no experiment has ruled out any of the interpretations of Module 6. The choice among them is driven by structural, conceptual, and philosophical considerations, not by empirical ones. But the structural constraints are nontrivial — the no-go theorems really do filter the space.
- **Bridge to Module 10.** The no-go theorems are proved in Module 10; this node states their consequences for interpretations without giving the full proofs. The HBAR-method principle "no interpretive rhetoric without formal grounding" means that any claim about what interpretations are or are not allowed must cite a theorem.
- **Motivation for formal foundations.** The empirical-equivalence situation is the reason formal foundations (Modules 10 and beyond) matter: they turn interpretational debate from a matter of taste into a matter of theorem. The no-go theorems are not philosophical arguments; they are mathematical results that any interpretation must respect or explicitly reject.
- **Underdetermination as a feature.** Rather than treating empirical equivalence as a scandal, one can read it as evidence that quantum mechanics has captured something *abstract* about the structure of physical prediction — something that can be realized in multiple concrete metaphysical ways. This is a Kuhnian / Friedman-style reading: the formalism is the stable core; the interpretations are the metaphysical clothing.

Mathematical Development

Bell's theorem — structural content

Consider an EPR-type scenario: Alice and Bob receive particles from a common source, each chooses a measurement setting $a, b \in \{0, 1\}$, and each obtains an outcome $A, B \in \{\pm 1\}$. Suppose the joint distribution $p(A, B|a, b)$ factorizes through a hidden variable λ:

$$p(A, B|a, b) = \int p(\lambda)\, p(A|a, \lambda)\, p(B|b, \lambda)\, d\lambda.$$

This is the local hidden-variable assumption: Alice's outcome depends only on her setting and λ, not on Bob's setting, and vice versa. The CHSH inequality

$$|E(0, 0) + E(0, 1) + E(1, 0) - E(1, 1)| \leq 2$$

(where $E(a, b) = \sum_{A,B} AB\, p(A, B|a, b)$) follows. Quantum mechanics predicts, and experiment confirms, CHSH values up to $2\sqrt{2}$. Therefore no local hidden-variable factorization is possible.

Consequences for interpretations:

- Copenhagen: no hidden variables, so the assumption does not apply. Consistent with Bell.
- Many-worlds: no definite outcomes, so the joint distribution $p(A, B|a, b)$ is not a single distribution but a branching structure. Consistent with Bell via denying the premise.
- Bohmian mechanics: explicitly nonlocal. Accepts Bell's conclusion.
- QBism: denies that there is an observer-independent joint distribution over Alice's and Bob's outcomes. Consistent with Bell via denying the premise.

Kochen–Specker theorem — structural content

Assume a hidden-variable theory assigns to every observable O a value $v(O)$, such that:

1. For any set of mutually commuting observables $\{O_i\}$, the values $\{v(O_i)\}$ satisfy the same functional relations as the observables themselves (if $O_3 = O_1 + O_2$, then $v(O_3) = v(O_1) + v(O_2)$).
2. The value $v(O)$ does not depend on which commuting set O is measured alongside.

Kochen and Specker showed that in Hilbert space dimension $d = 3$, a finite set of 117 rays can be constructed such that no valuation satisfying both conditions is consistent. Peres, Mermin, and others later gave more economical proofs. The conclusion: non-contextual hidden variables are impossible in $d \geq 3$.

Consequences for interpretations:

- Bohmian mechanics: contextual by construction (the spin value depends on the apparatus). Consistent with KS.
- Copenhagen: no pre-existing values, so the premise does not apply. Consistent.
- Many-worlds: no single-world values, same as Bell. Consistent.
- QBism: values are agent-beliefs, not observer-independent facts. Consistent.

PBR theorem — structural content

Assume a hidden-variable theory in which each quantum state $|\psi\rangle$ corresponds to a probability distribution $\mu_\psi(\lambda)$ over hidden variables λ. The theory is ψ-ontic if μ_{ψ_1} and μ_{ψ_2} have disjoint support for any two distinct pure states; ψ-epistemic if they can overlap. PBR showed that under a preparation-independence assumption (independently prepared systems have independent hidden variables), ψ-epistemic theories contradict quantum predictions for certain carefully chosen measurements. Therefore, subject to preparation independence, the quantum state must be ψ-ontic.

Consequences for interpretations:

- Bohmian mechanics: ψ-ontic (the wavefunction is a real physical field). Survives PBR.
- Many-worlds: ψ-ontic. Survives.
- Copenhagen: ψ-epistemic, but denies the premise that the wavefunction represents anything "behind the scenes." PBR applies only to theories that posit hidden variables λ beneath the wavefunction, which Copenhagen does not. Consistent.
- QBism: ψ-epistemic (the wavefunction is agent-belief), but the preparation independence assumption fails because beliefs are not physical objects that can be independently prepared. Consistent.

Free will and Frauchiger–Renner

The Free Will Theorem and the Frauchiger–Renner thought experiment further constrain interpretations, in ways that are interpretation-specific. Frauchiger–Renner in particular shows that *single-world* interpretations combined with quantum mechanics applied to reasoning observers can produce contradictions; many-worlds and QBism each dissolve the contradiction in their own ways, while Bohmian mechanics handles it by appeal to its specific single-world ontology.

Worked Example

Comparison table: interpretations versus no-go theorems

Interpretation	Bell (locality)	KS (non-contextuality)	PBR (ψ-epistemic)	Born rule derivation
Copenhagen	No hidden vars — evades	No hidden vars — evades	ψ-epistemic, no hiddens — evades	Axiomatic

Interpretation	Bell (locality)	KS (non-contextuality)	PBR (ψ-epistemic)	Born rule derivation
Many-worlds	No single-world outcomes — evades premise	No single-world values — evades premise	ψ-ontic — survives	Deutsch–Wallace / self-location
Bohmian mechanics	Nonlocal — accepts consequence	Contextual — accepts consequence	ψ-ontic — survives	Quantum equilibrium
Consistent histories	Framework-relative — evades premise	Framework-relative — evades premise	Framework-relative — survives	Decoherence functional
QBism	Agent-relative — evades premise	Agent-relative — evades premise	Agent-relative — survives (premise fails)	Dutch book

The pattern: most interpretations "evade" the no-go theorems by denying one of their premises. The exception is Bohmian mechanics, which is the only interpretation that directly faces Bell and KS and accepts their conclusions by being nonlocal and contextual respectively. Many-worlds, Copenhagen, and QBism each reject a premise (single-world outcomes, observer-independent values, or preparation independence), and the no-go theorems therefore do not apply in the form stated.

Scorecard on other axes

Ontological economy: - Copenhagen: minimal (one wavefunction, epistemic). - Many-worlds: minimal postulates but maximal ontology (all branches real). - Bohmian mechanics: wavefunction plus particles. - Consistent histories: framework choice is extra structure. - QBism: wavefunction relocated to agents; world is real but not described.

Resolution of measurement problem: - Copenhagen: denies the problem (unacceptable to realists). - Many-worlds: dissolves by unitary-only dynamics (ontologically costly). - Bohmian: dissolves by particle-definite positions (nonlocal). - CH: dissolves by framework relativity (framework proliferation). - QBism: dissolves by relocating the state (no objective world captured by ψ).

Compatibility with relativity: - Copenhagen: silent, relies on no-signaling. - Many-worlds: naturally relativistic (decoherence is local). - Bohmian: awkward; requires a preferred foliation. - CH: naturally relativistic. - QBism: naturally relativistic.

The underdetermination scandal dissolved

The fact that all five interpretations reproduce the same predictions is sometimes called the "interpretational underdetermination scandal." Structurally, it dissolves when one recognizes that the formalism of quantum mechanics is the stable, empirically validated core, and the interpretations are attempts to provide metaphysical narratives around it. The underdetermination is then analogous to the situation in classical mechanics, where one can choose to view the theory in Newtonian, Lagrangian, or Hamiltonian form — all mathematically equivalent, all empirically adequate, but suggesting different physical pictures. The difference is that in quantum mechanics the metaphysical differences between "equivalent" interpretations feel more dramatic, because they concern the nature of reality itself rather than just the choice of variables.

Cross-Links

- **Module 6.2–6.5 (Interpretations):** All four interpretations in this module satisfy empirical equivalence.
- **Module 10.1 (Bell's theorem):** Formal proof of the locality constraint.
- **Module 10.2 (Kochen–Specker):** Formal proof of the contextuality constraint.
- **Module 10.3 (PBR theorem):** Formal proof of the ψ-ontic constraint.

- **Module 10.4 (Ontological models):** The framework in which PBR and related results are formulated.
- **HBAR Method:** This node embodies the principle "no interpretive rhetoric without formal grounding." Every claim about which interpretations are allowed is backed by a theorem.

Structural Compression

Invariant: All interpretations developed in Module 6 are empirically equivalent to standard quantum mechanics. The choice among them is metaphysical, not empirical. But the metaphysical space is constrained by no-go theorems: Bell (locality), Kochen–Specker (non-contextuality), PBR (ψ-epistemic), which act as filters eliminating broad classes of hidden-variable theories.

Mechanism: Each no-go theorem proves the inconsistency of a specific set of assumptions with quantum statistics. Interpretations survive by denying one of the assumptions — nonlocality, contextuality, single-world ontology, preparation independence, or observer-independent joint distributions.

Cross-System Echo: Underdetermination of theory by data is generic in science, and the quantum-foundations case is the sharpest known example where the underdetermination is itself mathematically classified — every interpretive move corresponds to denying a specific premise of a specific theorem, and the theorems tell you which moves remain.

Exercises

1. State precisely what it means for two interpretations to be empirically equivalent. Why does this not imply they are "equally good"?
2. For each of Copenhagen, many-worlds, Bohmian, and QBism, identify the premise of Bell's theorem that the interpretation rejects or the consequence it accepts.
3. Explain, in structural terms, why Kochen–Specker does not apply to dimension $d = 2$ (single qubit). What is special about $d \geq 3$?
4. Summarize the PBR theorem's assumptions and conclusions in your own words. What would it mean for an interpretation to be "ψ-epistemic" in the PBR sense, and why does QBism escape the theorem's reach?
5. Produce a scorecard comparing the interpretations of Module 6 along three axes: ontological commitments, compatibility with relativity, and mechanism for the Born rule. Which axis do you find most decisive?
6. Describe a hypothetical experiment that could distinguish two interpretations of Module 6. If no such experiment exists within standard quantum mechanics, explain what would have to change (e.g., moving to objective-collapse theories, modified QM, etc.) for distinguishing experiments to become possible.
7. Argue, structurally, why the existence of no-go theorems turns interpretational debate into a disciplined enterprise rather than unconstrained speculation. What would foundations look like if no no-go theorems were known?