Empirical Equivalence and Constraints

Quantum Foundations · Module 6 — Interpretations

Node 6.6

Position in Knowledge Graph

System: Quantum Foundations **Structural Domain:** Interpretations **Node:** 6.6

The four interpretations developed in Module 6 — Copenhagen (6.2), many-worlds (6.3), Bohmian mechanics (6.4), and consistent histories / QBism (6.5) — are *empirically equivalent* in the precise sense that each reproduces the Born-rule predictions of standard quantum mechanics exactly. No experiment within the scope of standard quantum mechanics can distinguish them. And yet they make radically different claims about what exists: one wavefunction or many, with or without particles, with or without collapse, with or without framework relativity.

This node closes Module 6 by making the empirical-equivalence situation structurally precise and by identifying the no-go theorems that *do* constrain the space of admissible interpretations. The full formal content of these theorems is developed in Module 10, but their structural role — as filters on interpretive space — belongs here.

The punchline: no interpretation of standard quantum mechanics has been ruled out by experiment, and current no-go theorems eliminate only certain broad structural classes of hidden-variable theories. Choice of interpretation is therefore metaphysical, not empirical, but the metaphysical options are constrained rather than unconstrained.

Formal Statement

Empirical equivalence

Definition. Two interpretations I_1 and I_2 of standard quantum mechanics are *empirically equivalent* if, for every experimental setup and every quantum-mechanical measurement, the probability distribution over outcomes predicted by I_1 coincides exactly with that predicted by I_2 — and with the standard Born-rule prediction.

All four interpretations of Module 6 (and many related ones: relational quantum mechanics, modal interpretations, transactional, etc.) satisfy this definition with respect to standard non-relativistic quantum mechanics. So do objective-collapse theories (GRW, Penrose) to within current experimental precision, though these make predictions that differ from standard QM and are thus only *approximately* empirically equivalent.

The no-go theorems

The following three theorems (formalized in Module 10) act as structural filters on the space of interpretations:

Bell's theorem (Node 10.1). Any hidden-variable theory reproducing quantum statistics and assigning definite values to all measurement outcomes must be nonlocal: there must be a physical influence between spacelike-separated measurement events, in the form of dependence of one measurement's outcome on the setting of a distant measurement's apparatus. Formally: no local hidden-variable theory can reproduce the CHSH violations observed in Bell experiments.

Kochen–Specker theorem (Node 10.2). Any hidden-variable theory in Hilbert space dimension $d \geq 3$ that assigns pre-existing values to all observables must be *contextual*: the value assigned to an observable may depend on which other (commuting) observables are measured along with it. Non-contextual hidden variables are impossible.

PBR theorem (Node 10.3) — Pusey, Barrett, Rudolph (2012). Under the preparation independence assumption, any hidden-variable theory in which the quantum state is ψ-epistemic (i.e., two distinct pure states can correspond to overlapping probability distributions over hidden variables) contradicts quantum predictions. Therefore, subject to the premises, the quantum state must be ψ-ontic: distinct pure states correspond to disjoint ontic states.

Auxiliary results:

- **Free Will Theorem** (Conway–Kochen 2006, 2009). If experimenters have free will to choose measurement settings, then particles must also have an analogous "free will" (outcomes not determined by past history on the light cone). A strengthening of Bell.
- **Frauchiger–Renner** (2018). A thought experiment showing that single-world assumptions, quantum mechanics applied to observers, and certain reasoning principles jointly lead to contradictions. Each interpretation handles this differently.

Filter structure

Each theorem eliminates a specific class of interpretations:

- Bell eliminates **local hidden-variable theories.**
- Kochen–Specker eliminates **non-contextual hidden-variable theories** (in $d \geq 3$).
- PBR eliminates a large class of **ψ-epistemic hidden-variable theories.**

What remains is the space of admissible interpretations: nonlocal or contextual hidden-variable theories (Bohmian), ψ-ontic theories without hidden variables (many-worlds), or theories that reject one of the premises of the no-go theorems (QBism rejects the "observer-independent joint distribution" premise, and in that sense evades rather than satisfies the theorems).

Intuition

The empirical-equivalence situation is sometimes described as a scandal: how can multiple mutually exclusive accounts of reality all make the same predictions? The structural answer is that standard quantum mechanics is, itself, under-committal about ontology. It specifies a formalism (Hilbert space + Born rule) and tells you how to use it to predict experiments. The *interpretations* are different ways of filling in a metaphysical story that the formalism alone leaves open. Since the formalism determines the predictions and the predictions are what experiments measure, adding extra metaphysics on top of the formalism cannot change experimental outcomes.

But "empirically equivalent" does not mean "equally good" or "equally plausible." Interpretations can be compared along axes other than prediction:

- **Ontological economy.** How much extra machinery beyond the formalism does the interpretation require? Bohmian mechanics adds particle positions; many-worlds subtracts the collapse rule and claims branches are real; QBism relocates the wavefunction to the agent's head.
- **Conceptual clarity.** Does the interpretation dissolve the measurement problem, or merely relocate it?
- **Compatibility with relativity.** Bohmian mechanics is explicitly nonlocal and fits uneasily with Lorentz invariance. Many-worlds and QBism are more naturally relativistic.
- **Compatibility with cosmology.** Interpretations requiring an external observer or classical apparatus (Copenhagen) struggle with cosmological contexts where there is no such external structure.
- **Derivation of the Born rule.** Some interpretations take Born as axiomatic; others attempt to derive it from decision theory (Deutsch–Wallace), self-location (Sebens–Carroll), or Dutch-book arguments

(QBism).

The no-go theorems narrow this space. Before Bell (1964), it was possible to hope for a local classical picture behind quantum statistics; Bell proved this hope false. Before KS (1967), it was possible to hope for non-contextual hidden variables; KS proved this hope false in dimension ≥ 3. Before PBR (2012), it was possible to hope for a broad class of ψ-epistemic theories; PBR constrained them. Each theorem is a reduction in the space of viable stories.

What remains is: if you want classical hidden variables, you must accept nonlocality and contextuality (Bohmian mechanics does this). If you want to keep locality, you must give up single-world definite outcomes (many-worlds) or relocate the state to the agent (QBism). If you want to keep both locality and a single world, you must give up some other principle (e.g., counterfactual definiteness). There is no option that keeps everything.

Structural Role

- **Closing the module.** This node states clearly: no experiment has ruled out any of the interpretations of Module 6. The choice among them is driven by structural, conceptual, and philosophical considerations, not by empirical ones. But the structural constraints are nontrivial — the no-go theorems really do filter the space.
- **Bridge to Module 10.** The no-go theorems are proved in Module 10; this node states their consequences for interpretations without giving the full proofs. The HBAR-method principle "no interpretive rhetoric without formal grounding" means that any claim about what interpretations are or are not allowed must cite a theorem.
- **Motivation for formal foundations.** The empirical-equivalence situation is the reason formal foundations (Modules 10 and beyond) matter: they turn interpretational debate from a matter of taste into a matter of theorem. The no-go theorems are not philosophical arguments; they are mathematical results that any interpretation must respect or explicitly reject.
- **Underdetermination as a feature.** Rather than treating empirical equivalence as a scandal, one can read it as evidence that quantum mechanics has captured something *abstract* about the structure of physical prediction — something that can be realized in multiple concrete metaphysical ways. This is a Kuhnian / Friedman-style reading: the formalism is the stable core; the interpretations are the metaphysical clothing.

Mathematical Development

Bell's theorem — structural content

Consider an EPR-type scenario: Alice and Bob receive particles from a common source, each chooses a measurement setting $a, b \in \{0, 1\}$, and each obtains an outcome $A, B \in \{\pm 1\}$. Suppose the joint distribution $p(A, B|a, b)$ factorizes through a hidden variable λ:

$$p(A, B|a, b) = \int p(\lambda)\, p(A|a, \lambda)\, p(B|b, \lambda)\, d\lambda.$$

This is the local hidden-variable assumption: Alice's outcome depends only on her setting and λ, not on Bob's setting, and vice versa. The CHSH inequality

$$|E(0,0) + E(0,1) + E(1,0) - E(1,1)| \leq 2$$

(where $E(a, b) = \sum_{A,B} AB\, p(A, B|a, b)$) follows. Quantum mechanics predicts, and experiment confirms, CHSH values up to $2\sqrt{2}$. Therefore no local hidden-variable factorization is possible.

Consequences for interpretations:

- Copenhagen: no hidden variables, so the assumption does not apply. Consistent with Bell.
- Many-worlds: no definite outcomes, so the joint distribution $p(A, B|a, b)$ is not a single distribution but a branching structure. Consistent with Bell via denying the premise.
- Bohmian mechanics: explicitly nonlocal. Accepts Bell's conclusion.
- QBism: denies that there is an observer-independent joint distribution over Alice's and Bob's outcomes. Consistent with Bell via denying the premise.

Kochen–Specker theorem — structural content

Assume a hidden-variable theory assigns to every observable O a value $v(O)$, such that:

1. For any set of mutually commuting observables $\{O_i\}$, the values $\{v(O_i)\}$ satisfy the same functional relations as the observables themselves (if $O_3 = O_1 + O_2$, then $v(O_3) = v(O_1) + v(O_2)$).
2. The value $v(O)$ does not depend on which commuting set O is measured alongside.

Kochen and Specker showed that in Hilbert space dimension $d = 3$, a finite set of 117 rays can be constructed such that no valuation satisfying both conditions is consistent. Peres, Mermin, and others later gave more economical proofs. The conclusion: non-contextual hidden variables are impossible in $d \geq 3$.

Consequences for interpretations:

- Bohmian mechanics: contextual by construction (the spin value depends on the apparatus). Consistent with KS.
- Copenhagen: no pre-existing values, so the premise does not apply. Consistent.
- Many-worlds: no single-world values, same as Bell. Consistent.
- QBism: values are agent-beliefs, not observer-independent facts. Consistent.

PBR theorem — structural content

Assume a hidden-variable theory in which each quantum state $|\psi\rangle$ corresponds to a probability distribution $\mu_\psi(\lambda)$ over hidden variables λ. The theory is ψ-ontic if μ_{ψ_1} and μ_{ψ_2} have disjoint support for any two distinct pure states; ψ-epistemic if they can overlap. PBR showed that under a preparation-independence assumption (independently prepared systems have independent hidden variables), ψ-epistemic theories contradict quantum predictions for certain carefully chosen measurements. Therefore, subject to preparation independence, the quantum state must be ψ-ontic.

Consequences for interpretations:

- Bohmian mechanics: ψ-ontic (the wavefunction is a real physical field). Survives PBR.
- Many-worlds: ψ-ontic. Survives.
- Copenhagen: ψ-epistemic, but denies the premise that the wavefunction represents anything "behind the scenes." PBR applies only to theories that posit hidden variables λ beneath the wavefunction, which Copenhagen does not. Consistent.
- QBism: ψ-epistemic (the wavefunction is agent-belief), but the preparation independence assumption fails because beliefs are not physical objects that can be independently prepared. Consistent.

Free will and Frauchiger–Renner

The Free Will Theorem and the Frauchiger–Renner thought experiment further constrain interpretations, in ways that are interpretation-specific. Frauchiger–Renner in particular shows that *single-world* interpretations combined with quantum mechanics applied to reasoning observers can produce contradictions; many-worlds and QBism each dissolve the contradiction in their own ways, while Bohmian mechanics handles it by appeal to its specific single-world ontology.

Worked Example

Comparison table: interpretations versus no-go theorems

Interpretation	Bell (locality)	KS (non-contextuality)	PBR (ψ-epistemic)	Born rule derivation
Copenhagen	No hidden vars — evades	No hidden vars — evades	ψ-epistemic, no hiddens — evades	Axiomatic
Many-worlds	No single-world outcomes — evades premise	No single-world values — evades premise	ψ-ontic — survives	Deutsch–Wallace / self-location
Bohmian mechanics	Nonlocal — accepts consequence	Contextual — accepts consequence	ψ-ontic — survives	Quantum equilibrium
Consistent histories	Framework-relative — evades premise	Framework-relative — evades premise	Framework-relative — survives	Decoherence functional
QBism	Agent-relative — evades premise	Agent-relative — evades premise	Agent-relative — survives (premise fails)	Dutch book

The pattern: most interpretations "evade" the no-go theorems by denying one of their premises. The exception is Bohmian mechanics, which is the only interpretation that directly faces Bell and KS and accepts their conclusions by being nonlocal and contextual respectively. Many-worlds, Copenhagen, and QBism each reject a premise (single-world outcomes, observer-independent values, or preparation independence), and the no-go theorems therefore do not apply in the form stated.

Scorecard on other axes

Ontological economy: - Copenhagen: minimal (one wavefunction, epistemic). - Many-worlds: minimal postulates but maximal ontology (all branches real). - Bohmian mechanics: wavefunction plus particles. - Consistent histories: framework choice is extra structure. - QBism: wavefunction relocated to agents; world is real but not described.

Resolution of measurement problem: - Copenhagen: denies the problem (unacceptable to realists). - Many-worlds: dissolves by unitary-only dynamics (ontologically costly). - Bohmian: dissolves by particle-definite positions (nonlocal). - CH: dissolves by framework relativity (framework proliferation). - QBism: dissolves by relocating the state (no objective world captured by ψ).

Compatibility with relativity: - Copenhagen: silent, relies on no-signaling. - Many-worlds: naturally relativistic (decoherence is local). - Bohmian: awkward; requires a preferred foliation. - CH: naturally relativistic. - QBism: naturally relativistic.

The underdetermination scandal dissolved

The fact that all five interpretations reproduce the same predictions is sometimes called the "interpretational underdetermination scandal." Structurally, it dissolves when one recognizes that the formalism of quantum mechanics is the stable, empirically validated core, and the interpretations are attempts to provide metaphysical narratives around it. The underdetermination is then analogous to the situation in classical mechanics, where one can choose to view the theory in Newtonian, Lagrangian, or Hamiltonian form — all mathematically equivalent, all empirically adequate, but suggesting different physical pictures. The difference is that in quantum mechanics the metaphysical differences between "equivalent" interpretations feel more dramatic, because they concern the nature of reality itself rather than just the choice of variables.

Cross-Links

- **Module 6.2–6.5 (Interpretations):** All four interpretations in this module satisfy empirical equivalence.
- **Module 10.1 (Bell's theorem):** Formal proof of the locality constraint.
- **Module 10.2 (Kochen–Specker):** Formal proof of the contextuality constraint.
- **Module 10.3 (PBR theorem):** Formal proof of the ψ-ontic constraint.
- **Module 10.4 (Ontological models):** The framework in which PBR and related results are formulated.
- **HBAR Method:** This node embodies the principle "no interpretive rhetoric without formal grounding." Every claim about which interpretations are allowed is backed by a theorem.

Structural Compression

Invariant: All interpretations developed in Module 6 are empirically equivalent to standard quantum mechanics. The choice among them is metaphysical, not empirical. But the metaphysical space is constrained by no-go theorems: Bell (locality), Kochen–Specker (non-contextuality), PBR (ψ-epistemic), which act as filters eliminating broad classes of hidden-variable theories.

Mechanism: Each no-go theorem proves the inconsistency of a specific set of assumptions with quantum statistics. Interpretations survive by denying one of the assumptions — nonlocality, contextuality, single-world ontology, preparation independence, or observer-independent joint distributions.

Cross-System Echo: Underdetermination of theory by data is generic in science, and the quantum-foundations case is the sharpest known example where the underdetermination is itself mathematically classified — every interpretive move corresponds to denying a specific premise of a specific theorem, and the theorems tell you which moves remain.

Exercises

1. State precisely what it means for two interpretations to be empirically equivalent. Why does this not imply they are "equally good"?
2. For each of Copenhagen, many-worlds, Bohmian, and QBism, identify the premise of Bell's theorem that the interpretation rejects or the consequence it accepts.
3. Explain, in structural terms, why Kochen–Specker does not apply to dimension $d = 2$ (single qubit). What is special about $d \geq 3$?
4. Summarize the PBR theorem's assumptions and conclusions in your own words. What would it mean for an interpretation to be "ψ-epistemic" in the PBR sense, and why does QBism escape the theorem's reach?
5. Produce a scorecard comparing the interpretations of Module 6 along three axes: ontological commitments, compatibility with relativity, and mechanism for the Born rule. Which axis do you find most decisive?
6. Describe a hypothetical experiment that could distinguish two interpretations of Module 6. If no such experiment exists within standard quantum mechanics, explain what would have to change (e.g., moving to objective-collapse theories, modified QM, etc.) for distinguishing experiments to become possible.
7. Argue, structurally, why the existence of no-go theorems turns interpretational debate into a disciplined enterprise rather than unconstrained speculation. What would foundations look like if no no-go theorems were known?

Quantum Foundations · Module 6 — Interpretations · Node 6.6
Concepts: hypothesis-testing, entanglement
Prerequisites: 6.2, 6.3, 6.4, 6.5
Enables: 10.3