Open vs Closed Quantum Systems

Quantum Foundations · Module 8 — Open Systems

Node 8.1

Position in Knowledge Graph

System: Quantum Foundations **Structural Domain:** Open Quantum Systems **Node:** 8.1

This is the foundation node of Module 8. The dynamical postulate (Node 2.4) gave us unitary evolution for *closed* quantum systems. Real systems are never closed. This module formalizes the dynamics of systems that exchange energy, information, or particles with an environment — and shows how the unitary postulate is a special case, not the general one.

Formal Definition

A quantum system is **closed** if its dynamics are generated by a Hamiltonian acting on a fixed Hilbert space $\mathcal{H}$, producing the unitary evolution

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle, \qquad U(t) = e^{-iHt/\hbar}.$$

A quantum system is **open** if it is one part of a larger composite system whose other part — the **environment** — is inaccessible. The total composite system $S + E$ is closed and evolves unitarily on $\mathcal{H}_S \otimes \mathcal{H}_E$, but the **reduced state** of the system

$$\rho_S(t) = \mathrm{tr}_E \left(U(t)(\rho_S(0) \otimes \rho_E(0))U^\dagger(t) \right)$$

is in general **not** generated by any unitary on $\mathcal{H}_S$ alone.

The map $\rho_S(0) \mapsto \rho_S(t)$ is a quantum channel (Node 7.3), i.e., a CPTP map. The collection $\{\mathcal{E}_t\}_{t \geq 0}$ is the **dynamical map** of the open system.

Intuition

A closed system is a fiction. Every real system couples to *something* — vacuum fluctuations, thermal photons, gravitational fields, the experimentalist's apparatus. The closed-system idealization is useful for short times and well-isolated systems but it cannot describe the full physical situation.

The open-system formalism is the correction: instead of pretending isolation, we explicitly model the system + environment, evolve the joint system unitarily, then trace out the environment. What we are left with — the reduced density matrix evolution — is in general non-unitary, irreversible, and entropy-increasing.

This is the structural origin of:

- **Dissipation** (energy flowing from system to environment).
- **Decoherence** (quantum coherences leaking into environmental degrees of freedom).

- **Thermalization** (relaxation to equilibrium with the environment).
- **Measurement** (apparatus-as-environment).

Structural Role

This node sets up Module 8's central question: *what dynamical equation governs the reduced state $\rho_S(t)$?*

In general, the answer involves complicated integro-differential equations with memory effects. But under physically motivated assumptions — weak coupling, fast environment, no memory — the dynamics simplify dramatically to the **Lindblad master equation** (Node 8.2). This is the workhorse equation of open-system quantum mechanics and is what every quantum hardware noise model ultimately reduces to.

Module 8 then unpacks:

- The Lindblad form (8.2) — the differential generator of CPTP semigroups.
- Markovian vs non-Markovian dynamics (8.3) — when memory effects matter.
- Quantum trajectories (8.4) — stochastic unraveling of master equations.
- Decoherence-free subspaces (8.5) — protected sectors immune to noise.
- Quantum error correction foundations (8.6) — engineered restoration of coherence.

Mathematical Development

Why open-system dynamics are non-unitary

If the system + environment start in a *product* state $\rho_S \otimes \rho_E$ and the joint system evolves unitarily, the reduced state evolution is

$$\rho_S(t) = \mathrm{tr}_E \left(U(t)(\rho_S(0) \otimes \rho_E(0))U^\dagger(t) \right).$$

Even though U is unitary on the joint space, the partial trace introduces a contraction: distinct $\rho_S(0)$ can lead to the same $\rho_S(t)$, violating unitarity (which is invertible).

Concretely, this is the source of entropy increase: $S(\rho_S(t))$ can grow with time even though $S(\rho_{SE}(t)) = S(\rho_{SE}(0))$ is conserved by the unitary.

Markov approximation

If the environment is "large and fast" — i.e., correlations within the environment decay on timescales much shorter than the system's dynamics — then the dynamical map satisfies the **semigroup property**:

$$\mathcal{E}_{t+s} = \mathcal{E}_t \circ \mathcal{E}_s, \qquad t, s \geq 0.$$

A semigroup of CPTP maps is generated by a **Lindbladian** (Node 8.2). Without the semigroup property, the dynamics is non-Markovian (Node 8.3) and qualitatively richer.

When openness can be ignored

The closed-system approximation is valid when:

- Coupling g to the environment is weak compared to internal energy scales.
- Time of interest t is much shorter than $1/(g^2\tau_E)$, where τ_E is the environment correlation time.
- Environmental temperature is below the relevant excitation gap.

In quantum hardware, gate times are much shorter than T_1, T_2, but circuit depths are bounded by them — this is where the open-system approximation becomes operationally important.

Worked Example

Pure dephasing of a qubit

Take a qubit coupled to a bath via $H_{SE} = Z \otimes B$ for some environment operator B. The dynamical map turns out to be

$$\rho_S(t) = \begin{pmatrix} \rho_{00} & e^{-\Gamma(t)}\rho_{01} \\ e^{-\Gamma(t)}\rho_{10} & \rho_{11} \end{pmatrix},$$

with $\Gamma(t)$ a positive function determined by the environment correlation function. The diagonal (populations) is unchanged, but off-diagonal (coherences) decay. This is exactly the structural picture of decoherence: pointer states (here, $|0\rangle, |1\rangle$) survive, superpositions across them do not.

For a Markovian bath, $\Gamma(t) = \gamma t$ — exponential decay of coherences. For a non-Markovian bath, $\Gamma(t)$ can be non-monotonic, with revivals.

Cross-Links

- **Module 5:** Decoherence (5.3, 5.4) is the open-system perspective on coherence loss.
- **Module 7:** Quantum channels (7.3) are the discrete-time / time-integrated form of open-system dynamics.
- **VQA:** Hardware noise models live here. Noise-aware ansatz design and error mitigation depend on the structure of the open-system dynamics.
- **Quantum Thermodynamics (Module 9):** Thermal equilibration is open-system dynamics with a thermal environment.

Structural Compression

Invariant: Closed-system dynamics is unitary; open-system dynamics is CPTP. Unitarity is the special case in which the environment is trivial.

Mechanism: Tracing out an environment after joint unitary evolution.

Cross-System Echo: Coarse-graining in classical statistical mechanics; same structural operation produces irreversible dynamics from reversible underlying laws.

Exercises

1. Show that the dynamical map $\rho_S(0) \mapsto \rho_S(t)$ defined by tracing out the environment is CPTP.
2. Construct a 2-qubit system + environment such that the reduced dynamics on the system qubit has a specified Kraus form.
3. Explain why an initially pure state $\rho_S(0) = |\psi\rangle\langle\psi|$ can become mixed under open-system dynamics, and why this is not in conflict with the unitarity of the joint evolution.
4. Argue why the semigroup property requires the environment to be "memoryless," and give one physical example of a system in which this fails.
5. For the dephasing channel above, compute $\rho_S(t)$ starting from $|+\rangle$ and verify that it converges to $I/2$ as $t \to \infty$.

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Concepts: unitary-evolution, quantum-channels, density-matrix, decoherence
Prerequisites: 2.4, 5.3, 7.3
Enables: 8.2, 8.3

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