

Statistical Models

Statistics · Module 2 — Statistical Models

Node 2.1

Position in Knowledge Graph

System:

Statistics

Structural Domain:

Statistical Models and Likelihood

Node:

2.1

Formal Definition

A **statistical model** is a family of probability distributions:

$$\mathcal{P} = \{\mathbb{P}_\theta : \theta \in \Theta\}.$$

- Θ is the parameter space.
 - Each θ indexes a probability distribution.
 - Data is assumed to arise from one unknown member of this family.
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Intuition

Probability theory assumes the distribution is known.

Statistics assumes:

The distribution belongs to a structured family, but we do not know which one.

The parameter θ encodes the unknown structure of reality.

Inference means learning θ from data.

Structural Role

Statistical models:

- Bridge probability and data.
- Introduce parameters.
- Define the object of estimation.
- Enable likelihood construction.

Without models, data cannot inform unknown structure.

Mathematical Development

Parametric Model

Finite-dimensional parameter space:

$$\theta \in \mathbb{R}^k.$$

Example: Normal model

$$X_i \sim \mathcal{N}(\mu, \sigma^2).$$

$$\theta = (\mu, \sigma^2).$$

Nonparametric Model

Infinite-dimensional parameter space.

Example: All continuous distributions on $\mathbb{R}$.

Identifiability

A model is **identifiable** if:

$$\mathbb{P}_{\theta_1} = \mathbb{P}_{\theta_2} \Rightarrow \theta_1 = \theta_2.$$

Otherwise, parameters cannot be uniquely recovered.

Worked Example

Bernoulli Model

$$X_i \sim \text{Bernoulli}(p), \quad p \in [0, 1].$$

Parameter p fully determines the distribution.

Data informs p .

Cross-Links

- **Linear Algebra:** Parameter vectors as elements of $\mathbb{R}^k$.
 - **AI:** Model class in machine learning.
 - **Economics:** Structural models of agents.
 - **Quantum:** Quantum states parameterize measurement distributions.
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Structural Compression

Invariant:

Model = structured family of distributions.

Mechanism:

Unknown parameter indexes probability measure.

Cross-System Echo:

In ML, hypothesis class defines learning space.

Exercises

1. Define a Poisson statistical model.
 2. Give an example of a non-identifiable model.
 3. Explain difference between parametric and nonparametric models.
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Concepts: probability-space, random-variables

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