Feedback Mechanisms

Systems Thinking · Module 1 — Feedback Loops

Node 1.1

Position in Knowledge Graph

This node introduces the fundamental structural object of the course: the feedback mechanism.

It precedes: - Positive and Negative Feedback (1.2) - Formal Dynamical Systems (2.1)

Feedback is the ontological primitive from which recursive dynamics arise.

Formal Statement / Definition / Construction

Let $\mathcal{X} \subseteq \mathbb{R}^n$ be a state space.

A **feedback mechanism** is a tuple

$$\mathcal{F} = (\mathcal{X}, F)$$

where

$$F : \mathcal{X} \to \mathcal{X}$$

is a state-update mapping.

The induced discrete-time evolution is defined recursively by

$$x_{t+1} = F(x_t), \quad t \in \mathbb{N}.$$

Equivalently, a continuous-time feedback mechanism is defined by

$$\dot{x}(t) = f(x(t)),$$

where

$$f : \mathcal{X} \to \mathbb{R}^n$$

is a vector field.

In both cases, the defining structural property is:

$$\text{Output at time } t \text{ influences input at time } t+1.$$

Formally, the system is **self-referential**:

$$x_{t+1} \text{ depends on } x_t.$$

Assumptions and Conditions

1. The state space $\mathcal{X}$ is a nonempty subset of $\mathbb{R}^n$.
2. The mapping F is well-defined on $\mathcal{X}$.
3. For continuous-time systems, assume f is locally Lipschitz to guarantee local existence and uniqueness of solutions.

No assumptions are made regarding linearity, stability, or boundedness at this stage.

Proof or Mathematical Development

A feedforward system has the form

$$y = G(u),$$

where output y depends only on input u.

In contrast, a feedback system satisfies

$$u = H(y),$$

so that

$$y = G(H(y)).$$

Thus,

$$y = (G \circ H)(y).$$

Let

$$F = G \circ H.$$

Then the system reduces to a self-mapping:

$$y = F(y).$$

In discrete-time evolution,

$$x_{t+1} = F(x_t)$$

is obtained by iterating this self-mapping.

Thus, feedback induces a dynamical system through function iteration.

The structure is recursive:

$$x_{t+k} = F^k(x_t),$$

where

$$F^k = \underbrace{F \circ F \circ \cdots \circ F}_{k \text{ times}}.$$

This recursive iteration is the mathematical engine generating trajectories.

Structural Role

Feedback mechanisms introduce:

- State recursion
- Temporal dependence
- Self-reference
- Potential for amplification or regulation

All later domains—dynamics, stability, control, networks, and emergence—require the existence of recursive state evolution.

Without feedback, no trajectory exists.

Feedback is the minimal structure that produces time-evolution.

Structural Compression

Invariant

A feedback mechanism is a self-mapping:

$$F : \mathcal{X} \to \mathcal{X}$$

whose iteration defines system evolution.

Mechanism

Recursive composition:

$$x_{t+1} = F(x_t), \quad x_{t+k} = F^k(x_t).$$

Cross-System Echo

- Linear Algebra: eigenvalues govern iteration of linear maps.
- Control Theory: closed-loop systems are feedback compositions.
- Economics: expectations feedback into price formation.
- Biology: regulatory networks are state-dependent update rules.

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Concepts: feedback, state-space, dynamical-systems
Enables: 1.2, 2.1

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