Causal Loop Diagrams

Systems Thinking · Module 1 — Feedback Loops

Node 1.3

Position in Knowledge Graph

This node formalizes causal loop diagrams (CLDs) as graphical representations of feedback structure.

It builds on: - Feedback mechanisms (1.1) - Positive and negative feedback (1.2)

It prepares for: - Stock-and-flow modeling (1.4) - Graph-theoretic network analysis (4.1)

Formal Statement / Definition / Construction

Let $V = \{x_1, \dots, x_n\}$ be a finite set of system variables.

A **causal loop diagram (CLD)** is a signed directed graph

$$\mathcal{G} = (V, E, \sigma),$$

where:

- $E \subseteq V \times V$ is a set of directed edges,
- $\sigma : E \to \{+1, -1\}$ assigns a sign to each edge.

An edge $(x_i, x_j) \in E$ with sign σ_{ij} represents a causal relation:

$$\frac{\partial x_j}{\partial x_i} \text{ has sign } \sigma_{ij}.$$

Interpretation:

- $\sigma_{ij} = +1$: an increase in x_i tends to increase x_j (all else equal).
- $\sigma_{ij} = -1$: an increase in x_i tends to decrease x_j.

Definition (Feedback Loop)

A **feedback loop** is a directed cycle

$$x_{i_1} \to x_{i_2} \to \cdots \to x_{i_k} \to x_{i_1}$$

in $\mathcal{G}$.

The **loop sign** is defined as

$$\Sigma = \prod_{m=1}^{k} \sigma_{i_m i_{m+1}},$$

with indices taken modulo k.

- $\Sigma = +1$: reinforcing (positive) loop.
- $\Sigma = -1$: balancing (negative) loop.

Assumptions and Conditions

1. The variable set V is finite.
2. Signs encode local monotonic influence only.
3. No quantitative magnitude is assumed.
4. Causal relations are interpreted structurally, not statistically.
5. The CLD does not specify functional form.

CLDs represent qualitative structure, not dynamical equations.

Proof or Mathematical Development

Suppose the system admits a differentiable representation

$$\dot{x}_i = f_i(x_1, \ldots, x_n), \quad i = 1, \ldots, n.$$

Define the Jacobian matrix

$$J(x) = \left(\frac{\partial f_i}{\partial x_j} \right).$$

Then a CLD corresponds to the sign structure of J:

$$\sigma_{ji} = \operatorname{sign} \left(\frac{\partial f_i}{\partial x_j} \right),$$

whenever this derivative is nonzero.

Thus, a CLD abstracts the sign pattern of the system Jacobian.

For a feedback loop L, the product

$$\Sigma = \prod_{(i,j) \in L} \operatorname{sign} \left(\frac{\partial f_j}{\partial x_i} \right)$$

captures whether perturbations are amplified or counteracted along that cycle.

In linear systems

$$\dot{x} = Ax,$$

the CLD corresponds to the sign structure of matrix A.

Thus, causal loop diagrams are qualitative projections of underlying dynamical systems.

Structural Role

Causal loop diagrams:

- Encode feedback topology.
- Identify reinforcing and balancing cycles.
- Provide a bridge between qualitative reasoning and formal dynamics.
- Serve as the precursor to adjacency matrices and graph-theoretic analysis.

They do not define trajectories. They define interaction structure.

Dynamics arise only after functional forms are specified.

Structural Compression

Invariant

A causal loop diagram is a signed directed graph:

$$\mathcal{G} = (V, E, \sigma).$$

Mechanism

Feedback loops correspond to directed cycles; loop sign is the product of edge signs.

Cross-System Echo

- Linear Algebra: sign structure of the Jacobian matrix.
- Network Theory: adjacency matrices and cycle detection.
- Control Theory: feedback interconnection topology.
- Economics: reinforcing vs stabilizing policy loops.

Systems Thinking · Module 1 — Feedback Loops · Node 1.3
Concepts: feedback, graph-theory, adjacency-matrix
Prerequisites: 1.1, 1.2
Enables: 1.4, 4.1

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