Cross-Scale Dynamics

Systems Thinking · Module 10 — Meta Systems

Node 10.2

Position in Knowledge Graph

This node formalizes cross-scale dynamics: coupled evolution across hierarchical levels where macro-states constrain micro-dynamics and micro-aggregates update macro-states.

It builds on: - Hierarchical Structure (10.1)

It prepares for: - Meta-Level Feedback (10.3) - Meta-System Transitions (10.4)

Cross-scale dynamics is the mathematical core of multi-level systems thinking.

Formal Statement / Definition / Construction

Let $L \geq 2$ levels with states $x_t^{(\ell)} \in \mathcal{X}^{(\ell)}$.

Define two operators for each $\ell = 1, \ldots, L-1$:

- Upward aggregation:
$$G^{(\ell)} : \mathcal{X}^{(\ell)} \to \mathcal{X}^{(\ell+1)}, \quad x^{(\ell+1)} = G^{(\ell)}(x^{(\ell)}).$$
- Downward constraint / parameterization:
$$H^{(\ell+1)} : \mathcal{X}^{(\ell+1)} \to \mathcal{P}^{(\ell)},$$
where $\mathcal{P}^{(\ell)}$ is a parameter space controlling level-ℓ dynamics.

Let micro-level dynamics depend on macro-induced parameters:

$$x_{t+1}^{(\ell)} = F^{(\ell)}\big(x_t^{(\ell)}, \pi_t^{(\ell)}\big), \quad \pi_t^{(\ell)} = H^{(\ell+1)}(x_t^{(\ell+1)}).$$

Macro-level updates depend on aggregated micro-state:

$$x_{t+1}^{(\ell+1)} = F^{(\ell+1)}\big(x_t^{(\ell+1)}, G^{(\ell)}(x_t^{(\ell)})\big).$$

A **cross-scale dynamical system** is the coupled recursion across all levels:

$$\begin{cases} x_{t+1}^{(1)} = F^{(1)}(x_t^{(1)}, H^{(2)}(x_t^{(2)})), \\ x_{t+1}^{(2)} = F^{(2)}(x_t^{(2)}, G^{(1)}(x_t^{(1)}), H^{(3)}(x_t^{(3)})), \\ \vdots \\ x_{t+1}^{(L)} = F^{(L)}(x_t^{(L)}, G^{(L-1)}(x_t^{(L-1)})). \end{cases}$$

Assumptions and Conditions

1. Each $\mathcal{X}^{(\ell)}$ is nonempty.
2. Maps $F^{(\ell)}, G^{(\ell)}, H^{(\ell)}$ are well-defined.
3. Updates are time-ordered to avoid algebraic loops at the same time index (i.e., x_{t+1} depends on x_t).
4. Aggregation $G^{(\ell)}$ is many-to-one in general (coarse-graining).
5. Parameterization $H^{(\ell+1)}$ may restrict feasible micro-actions by mapping macro-state into micro-parameter space.

No assumption is made about linearity or stability.

Proof or Mathematical Development

1. Reduction to Single Augmented System

Define augmented state:

$$X_t = (x_t^{(1)}, x_t^{(2)}, \dots, x_t^{(L)}) \in \mathcal{X}^{(1)} \times \cdots \times \mathcal{X}^{(L)}.$$

Define global update map $\mathcal{F}$ by the coupled recursion above, so:

$$X_{t+1} = \mathcal{F}(X_t).$$

Thus cross-scale dynamics is an ordinary dynamical system on the product space; "cross-scale" is a structural decomposition specifying which components influence which.

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2. Timescale Separation as a Special Case

A common regime is macro slower than micro.

Write micro with step size 1 and macro with step size $\epsilon \ll 1$ in continuous time:

$$\dot{x}^{(1)} = f^{(1)}(x^{(1)}, \pi), \quad \dot{x}^{(2)} = \epsilon f^{(2)}(x^{(2)}, G^{(1)}(x^{(1)})).$$

In discrete time, represent as:

$$\begin{aligned} x_{t+1}^{(1)} &= F^{(1)}(x_t^{(1)}, \pi_t), \\ x_{t+1}^{(2)} &= x_t^{(2)} + \epsilon \, \tilde{F}^{(2)}(x_t^{(2)}, G^{(1)}(x_t^{(1)})). \end{aligned}$$

This expresses slow macro drift under fast micro fluctuations.

Full singular perturbation results are deferred (requires additional analytic machinery).

Deferred: rigorous convergence of timescale-separated systems (singular perturbation / averaging theory).

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3. Cross-Scale Consistency Condition

If macro-state is defined purely by aggregation:

$$x_t^{(\ell+1)} = G^{(\ell)}(x_t^{(\ell)}),$$

then cross-scale closure requires that the macro update is consistent:

$$G^{(\ell)}(x_{t+1}^{(\ell)}) = F^{(\ell+1)}\big(G^{(\ell)}(x_t^{(\ell)})\big)$$

for all relevant $x_t^{(\ell)}$.

If this identity fails, then macro-dynamics is not an exact closure of micro-dynamics; it is an approximation. This explicitly distinguishes:

- exact coarse-graining (rare),
- approximate reduced models (common).

Structural Role

Cross-scale dynamics:

- Formalizes micro–macro coupling as bidirectional maps.
- Makes explicit the difference between aggregation and parameterization.
- Provides the object needed to define meta-feedback and meta-transitions.
- Enables disciplined talk about "top-down constraints" and "bottom-up emergence" without ambiguity.

This node is the rigorous bridge from hierarchical structure to meta-level feedback loops.

Structural Compression

Invariant

Cross-scale dynamics is a coupled recursion on levels:

$$x_{t+1}^{(\ell)} = F^{(\ell)}(x_t^{(\ell)}, H^{(\ell+1)}(x_t^{(\ell+1)})), \quad x_{t+1}^{(\ell+1)} = F^{(\ell+1)}(x_t^{(\ell+1)}, G^{(\ell)}(x_t^{(\ell)})).$$

Mechanism

Bidirectional coupling: upward aggregation G and downward parameterization H.

Cross-System Echo

- Control Theory: hierarchical and multi-rate control.
- Physics: renormalization and coarse-graining maps.
- Economics: microfoundations with macro policy constraints.
- Biology: organism-level regulation constraining cellular dynamics.

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Concepts: hierarchical-systems, dynamical-systems, feedback
Prerequisites: 10.1
Enables: 10.3, 10.4

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