Signal Propagation

Systems Thinking · Module 7 — Information Flow

Node 7.3

Position in Knowledge Graph

This node formalizes how signals and perturbations propagate through networks and dynamical systems.

It builds on: - Graph Foundations (4.1) - Shannon Entropy (7.1) - Channel Capacity (7.2)

It prepares for: - Information Integration (7.4) - Information Constraints in Systems (7.5)

Signal propagation connects topology, dynamics, and information flow.

Formal Statement / Definition / Construction

Let $G = (V, E)$ be a directed graph with adjacency matrix $A \in \mathbb{R}^{n\times n}$.

Consider linear network dynamics:

$$\dot{x}(t) = Ax(t) + Bu(t),$$

where:

- $x(t) \in \mathbb{R}^n$ is node state vector,
- $u(t) \in \mathbb{R}^m$ is external input,
- $B \in \mathbb{R}^{n\times m}$ is input matrix.

Impulse Response

Let $u(t) = \delta(t)e_k$ (impulse at node k).

Then response:

$$x(t) = e^{At}Be_k.$$

Thus propagation governed by matrix exponential e^{At}.

Path Expansion

Using series expansion:

$$e^{At} = \sum_{j=0}^{\infty} \frac{(At)^j}{j!}.$$

Hence:

$$x(t) = \sum_{j=0}^{\infty} \frac{t^j}{j!} A^j B e_k.$$

Term A^j corresponds to propagation along length-j paths.

Assumptions and Conditions

1. A constant (time-invariant).
2. System is well-posed (matrix exponential defined).
3. Network finite.
4. For probabilistic propagation, assume noise-free linear channel.

Nonlinear propagation requires separate treatment.

Proof or Mathematical Development

Propagation Speed in Linear Diffusion

Consider consensus-type dynamics:

$$\dot{x} = -Lx,$$

where L is graph Laplacian.

Solution:

$$x(t) = e^{-Lt} x(0).$$

Since Laplacian eigenvalues satisfy:

$$0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n,$$

convergence rate determined by spectral gap:

$$\lambda_2.$$

Thus information mixing time approximately:

$$\tau \sim \frac{1}{\lambda_2}.$$

Entropy Flow Interpretation

Let $X(t)$ denote random state.

Entropy evolution:

$$\frac{d}{dt}H(X(t)) = -\sum_x \dot{p}(x,t)\log p(x,t).$$

Under diffusion-like dynamics, entropy typically increases toward equilibrium.

Thus signal disperses over network.

Structural Role

Signal propagation:

- Links adjacency structure to dynamic influence.
- Determines speed of synchronization and diffusion.
- Connects spectral properties to mixing times.
- Provides quantitative model for information spread.

Propagation is governed by eigenstructure of network operator.

Structural Compression

Invariant

Signal response governed by:

$$x(t) = e^{At}x(0).$$

Mechanism

Matrix exponential expansion encodes path-based propagation:

$$e^{At} = \sum_{j=0}^{\infty} \frac{A^j t^j}{j!}.$$

Cross-System Echo

- Physics: heat equation diffusion.
- Economics: shock transmission across markets.
- Biology: neural signal propagation.
- Networks: rumor spreading dynamics.

Systems Thinking · Module 7 — Information Flow · Node 7.3
Concepts: graph-theory, adjacency-matrix, channel-capacity, dynamical-systems
Prerequisites: 4.1, 7.1, 7.2
Enables: 7.4, 7.5

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