Information Integration

Systems Thinking · Module 7 — Information Flow

Node 7.4

Position in Knowledge Graph

This node formalizes how distributed components combine information into coherent collective structure.

It builds on: - Shannon Entropy (7.1) - Signal Propagation (7.3)

It prepares for: - Information Constraints in Systems (7.5)

Information integration quantifies the degree to which system components form a unified informational structure.

Formal Statement / Definition / Construction

Let $X = (X_1, \ldots, X_n)$ be a collection of discrete random variables.

Joint entropy:

$$H(X) = H(X_1, \ldots, X_n).$$

Define total independent entropy:

$$H_{\text{ind}} = \sum_{i=1}^{n} H(X_i).$$

Definition (Multi-Information)

The **multi-information** (total correlation) is:

$$I_{\text{tot}}(X) = \sum_{i=1}^{n} H(X_i) - H(X_1, \ldots, X_n).$$

Equivalently:

$$I_{\text{tot}}(X) = D\left(p(x_1, \ldots, x_n) \parallel \prod_{i=1}^{n} p(x_i)\right),$$

where $D(\cdot\|\cdot)$ is KL divergence.

Definition (Integrated Information — Structural Form)

For a bipartition $\{A, B\}$ of indices:

$$\Phi_{A|B} = I(X_A; X_B).$$

System exhibits integration if:

$$\min_{A|B} \Phi_{A|B} > 0.$$

This measures minimal mutual dependence across partitions.

Assumptions and Conditions

1. Variables discrete with finite support.
2. Joint distribution well-defined.
3. Partition is nontrivial (both subsets nonempty).
4. Logarithm base fixed.

No claim about specific physical interpretation beyond informational dependence.

Proof or Mathematical Development

Non-Negativity

Since:

$$I_{\text{tot}}(X) = D\left(p(x) \parallel \prod_i p(x_i)\right),$$

and KL divergence satisfies:

$$D(p\|q) \geq 0,$$

we have:

$$I_{\text{tot}}(X) \geq 0.$$

Equality holds iff variables are mutually independent.

Decomposition

For $n = 2$:

$$I_{\text{tot}}(X_1, X_2) = H(X_1) + H(X_2) - H(X_1, X_2) = I(X_1; X_2).$$

For larger n, multi-information generalizes pairwise mutual information.

Interpretation in Networked Systems

If network state X arises from coupling:

$$\dot{x} = F(x),$$

then interaction induces statistical dependence among components.

Thus:

$$I_{\text{tot}}(X) > 0$$

reflects informational integration induced by coupling.

Structural Role

Information integration:

- Quantifies collective informational cohesion.
- Distinguishes independent subsystems from unified systems.
- Connects entropy to network coupling.
- Provides bridge to distributed computation and consciousness theories (formal aspects only).

It measures how much the whole exceeds sum of parts in informational terms.

Structural Compression

Invariant

Multi-information:

$$I_{\text{tot}}(X) = \sum_i H(X_i) - H(X).$$

Mechanism

KL divergence between joint distribution and product of marginals.

Cross-System Echo

- Physics: correlation functions.
- Biology: coordinated neural activity.
- Economics: systemic dependence.
- Networks: coupling-induced coherence.

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Concepts: shannon-entropy, mutual-information, information-theory
Prerequisites: 7.1, 7.3
Enables: 7.5

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