Information Constraints in Systems

Systems Thinking · Module 7 — Information Flow

Node 7.5

Position in Knowledge Graph

This node formalizes fundamental limits imposed by information theory on control, coordination, and adaptation.

It builds on: - Channel Capacity (7.2) - Information Integration (7.4)

It prepares for: - Institutions as Constraint Systems (8.1)

Information constraints determine what systems can know, transmit, and coordinate.

Formal Statement / Definition / Construction

Let a dynamical system be:

$$\dot{x}(t) = f(x(t), u(t)),$$

where:

- $x(t) \in \mathbb{R}^n$,
- $u(t)$ is control input,
- control is transmitted through a noisy channel.

Let communication channel have capacity:

$$C = \max_{p(x)} I(X;Y).$$

Theorem (Data-Rate Constraint for Stabilization — Informal Statement)

For a linear unstable system:

$$\dot{x} = Ax + Bu,$$

with unstable eigenvalues λ_i satisfying $\mathrm{Re}(\lambda_i) > 0$,

a necessary condition for stabilization over a noiseless digital channel of capacity C is:

$$C \geq \sum_{\mathrm{Re}(\lambda_i) > 0} \mathrm{Re}(\lambda_i).$$

This bound is known as the data-rate theorem.

Assumptions and Conditions

1. System is linear time-invariant.
2. Control signals transmitted over digital channel.
3. Channel has finite capacity C.
4. Stabilization means:

$$\lim_{t \to \infty} x(t) = 0.$$

5. Eigenvalues counted with multiplicity.

Full proof requires stochastic control theory; only structural derivation given.

Proof or Mathematical Development

Instability Growth Rate

For unstable mode:

$$x_i(t) \sim e^{\lambda_i t}.$$

To counteract instability, controller must receive state information at sufficient rate.

Information required to describe state with precision ϵ scales as:

$$\log \frac{1}{\epsilon}.$$

If state grows as $e^{\lambda t}$, precision requirement grows as:

$$\lambda t.$$

Thus minimum information rate required:

$$\frac{d}{dt}(\lambda t) = \lambda.$$

Summing over unstable modes yields lower bound:

$$C \geq \sum_{\mathrm{Re}(\lambda_i) > 0} \mathrm{Re}(\lambda_i).$$

If channel rate is below this threshold, controller cannot transmit corrective information fast enough.

Information–Integration Constraint

Let system partitioned into subsystems A, B.

If:

$$I(X_A; X_B) = 0,$$

then subsystems evolve independently.

Thus coordination across partitions requires:

$$I(X_A; X_B) > 0.$$

Limited channel capacity bounds achievable integration:

$$I(X_A; X_B) \leq C.$$

Structural Role

Information constraints:

- Bound achievable control performance.
- Limit coordination across subsystems.
- Connect eigenvalue instability to data rate.
- Formalize tradeoff between dynamics and communication.

Systems cannot overcome instability without sufficient informational throughput.

Structural Compression

Invariant

Minimum rate for stabilization:

$$C \geq \sum_{\mathrm{Re}(\lambda_i)>0} \mathrm{Re}(\lambda_i).$$

Mechanism

Instability generates exponential growth of uncertainty requiring proportional information rate.

Cross-System Echo

- Control Theory: data-rate theorem.
- Economics: limits of information aggregation.
- Biology: signaling bandwidth constraints.
- Networks: throughput limits in distributed coordination.

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Concepts: channel-capacity, control-theory, information-theory, constrained-optimization
Prerequisites: 7.2, 7.4
Enables: 8.1