

Institutions as Constraint Systems

Systems Thinking · Module 8 — Institutional Systems

Node 8.1

Position in Knowledge Graph

This node formalizes institutions as structured constraint systems imposed on agent behavior.

It builds on: - Information Constraints in Systems (7.5)

It prepares for: - Norms and Enforcement (8.2) - Constraint Architecture (9.1)

Institutions restrict feasible actions and shape collective dynamics.

Formal Statement / Definition / Construction

Let there be n agents.

Each agent i has:

- State $x_i \in \mathcal{X}_i$,
- Action space $\mathcal{A}_i$,
- Utility function $U_i : \mathcal{X} \times \mathcal{A} \rightarrow \mathbb{R}$.

Let joint action space:

$$\mathcal{A} = \prod_{i=1}^n \mathcal{A}_i.$$

Definition (Institution)

An **institution** is a constraint mapping:

$$\mathcal{I} : \mathcal{A} \rightarrow \{0, 1\},$$

where:

$$\mathcal{I}(a) = 1$$

indicates action profile a is permitted.

Feasible action set:

$$\mathcal{A}^{\mathcal{I}} = \{a \in \mathcal{A} : \mathcal{I}(a) = 1\}.$$

Institutional Dynamics

Agent dynamics under institution:

$$\dot{x}_i = f_i(x, a), \quad a \in \mathcal{A}^{\mathcal{I}}.$$

Institution modifies feasible trajectories by restricting admissible actions.

Assumptions and Conditions

1. Action space finite or measurable.
2. Constraint mapping well-defined.
3. Agents select actions from feasible set.
4. Institution is exogenous at this stage.

No assumption of optimality or enforcement mechanism yet.

Proof or Mathematical Development

Constraint-Induced Feasible Set Reduction

Without institution:

$$a \in \mathcal{A}.$$

With institution:

$$a \in \mathcal{A}^{\mathcal{I}} \subseteq \mathcal{A}.$$

Thus feasible trajectory set:

$$\mathcal{T}^{\mathcal{I}} \subseteq \mathcal{T}.$$

Institution alters reachable states.

Stability Under Constraints

Let x^* be equilibrium without institution.

If x^* requires action $a^* \notin \mathcal{A}^{\mathcal{I}}$, then equilibrium infeasible.

Thus institutional constraints modify equilibrium structure.

Information Constraint Interaction

Let information flow between agents be bounded by capacity C .

Institution may enforce reporting rule:

$$a_i = g_i(\text{message}_i).$$

If message entropy:

$$H(\text{message}_i) \leq C,$$

then feasible coordination limited by communication constraint.

Thus institutions operate under information limits.

Structural Role

Institutions:

- Restrict action space.
- Modify reachable states.
- Shape equilibrium outcomes.
- Mediate coordination under information constraints.

They transform decentralized dynamics via rule imposition.

Structural Compression

Invariant

Institution = constraint:

$$\mathcal{A}^I \subseteq \mathcal{A}.$$

Mechanism

Restriction of feasible action profiles alters system trajectories.

Cross-System Echo

- Control Theory: admissible control sets.
 - Economics: legal and regulatory constraints.
 - Biology: regulatory gene networks.
 - Networks: protocol rules limiting message exchange.
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Concepts: institutions, constrained-optimization, nash-equilibrium

Prerequisites: 7.5

Enables: 8.2, 9.1

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