Path Dependence

Systems Thinking · Module 8 — Institutional Systems

Node 8.3

Position in Knowledge Graph

This node formalizes how historical trajectories constrain future states in institutional systems.

It builds on: - Bifurcations (2.4) - Norms and Enforcement (8.2)

It prepares for: - Collective Action (8.4) - Incentive Compatibility (9.3)

Path dependence captures history-sensitive equilibrium selection.

Formal Statement / Definition / Construction

Let state space $\mathcal{X} \subseteq \mathbb{R}^n$.

Consider a stochastic dynamical system:

$$x_{t+1} = F(x_t) + \varepsilon_t,$$

where ε_t is random perturbation.

Suppose system admits multiple stable equilibria:

$$x_1^*, \ldots, x_k^*.$$

Definition (Path Dependence)

The system exhibits **path dependence** if long-run outcome depends on initial condition or realized history of shocks.

Formally, let:

$$\mathbb{P}_{x_0}\left(\lim_{t\to\infty} x_t = x_i^*\right)$$

depend nontrivially on x_0.

Equivalently:

$$\exists x_0 \neq x_0' \quad \text{such that} \quad \mathbb{P}_{x_0}(x_i^*) \neq \mathbb{P}_{x_0'}(x_i^*).$$

Assumptions and Conditions

1. Multiple attractors exist.
2. Basin of attraction boundaries nontrivial.
3. Noise term has bounded variance.
4. Transition probabilities well-defined.

If system has unique globally stable equilibrium, path dependence absent.

Proof or Mathematical Development

Deterministic Case

Consider:

$$\dot{x} = f(x),$$

with two stable equilibria x_1^*, x_2^*.

Let basin of attraction:

$$\mathcal{B}_i = \{x_0 : \lim_{t \to \infty} x(t) = x_i^*\}.$$

If:

$$\mathcal{B}_1 \neq \emptyset \quad \text{and} \quad \mathcal{B}_2 \neq \emptyset,$$

then final state depends on initial condition.

Stochastic Reinforcement Example

Let proportion $p_t \in [0, 1]$ evolve as:

$$p_{t+1} = p_t + \frac{1}{t}(X_t - p_t),$$

where:

$$\mathbb{P}(X_t = 1) = p_t.$$

This is Polya urn process.

Limit:

$$p_t \to P,$$

where P is random variable distributed on [0,1].

Outcome depends on early realizations.

Thus system locks into path-specific equilibrium.

Lock-In Mechanism

Suppose payoff increases with adoption:

$$U(a) = \alpha a + \beta a^2.$$

Increasing returns ($\beta > 0$) amplify early differences.

Small historical advantage becomes permanent dominance.

Structural Role

Path dependence:

- Explains institutional lock-in.
- Links increasing returns to equilibrium multiplicity.
- Connects bifurcation theory to historical processes.
- Formalizes irreversibility in social systems.

History shapes feasible equilibrium selection.

Structural Compression

Invariant

Multiple basins of attraction:

$$\mathcal{B}_i \neq \emptyset.$$

Mechanism

Positive feedback amplifies early deviations.

Cross-System Echo

- Economics: technology lock-in.
- Biology: evolutionary contingency.
- Physics: symmetry breaking.
- Networks: adoption cascades.

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Concepts: path-dependence, bifurcation, dynamical-systems, stability
Prerequisites: 2.4, 8.2
Enables: 8.4, 9.3

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