

Collective Action

Systems Thinking · Module 8 — Institutional Systems

Node 8.4

Position in Knowledge Graph

This node formalizes coordination problems where individually rational behavior conflicts with group-optimal outcomes.

It builds on: - Norms and Enforcement (8.2) - Path Dependence (8.3)

It prepares for: - Institutional Stability (8.5) - Principal-Agent Theory (9.2)

Collective action problems arise when public goods or shared resources are involved.

Formal Statement / Definition / Construction

Let there be n agents.

Each agent chooses action:

$$a_i \in \{0, 1\},$$

where:

- $a_i = 1$: contribute,
- $a_i = 0$: do not contribute.

Let total contributions:

$$S = \sum_{i=1}^n a_i.$$

Each agent has utility:

$$U_i(a) = B(S) - ca_i,$$

where:

- $B(S)$ is benefit function, increasing in S ,
 - $c > 0$ is private contribution cost.
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Assumptions and Conditions

1. Benefit function satisfies:

$$B'(S) > 0.$$

2. Benefits are non-excludable.
 3. Each agent maximizes own utility.
 4. Actions chosen simultaneously.
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Proof or Mathematical Development

Individual Incentive

Agent i 's marginal benefit of contributing:

$$\Delta U_i = B(S_{-i} + 1) - B(S_{-i}) - c.$$

If:

$$B(S_{-i} + 1) - B(S_{-i}) < c,$$

then contribution not individually rational.

Social Optimum

Social welfare:

$$W(a) = \sum_{i=1}^n U_i(a) = nB(S) - cS.$$

Marginal social benefit:

$$\Delta W = n[B(S + 1) - B(S)] - c.$$

Thus social optimum may require:

$$n[B(S + 1) - B(S)] > c,$$

even if:

$$B(S + 1) - B(S) < c.$$

Hence:

Individual incentive $\neq$ Social optimum.

Free-Rider Equilibrium

If for all S_{-i} :

$$B(S_{-i} + 1) - B(S_{-i}) < c,$$

then Nash equilibrium:

$$a_i = 0 \quad \forall i.$$

Public good underprovided.

Institutional Remedy

Introduce sanction s for non-contribution.

Effective utility:

$$\tilde{U}_i(a) = B(S) - ca_i - s(1 - a_i).$$

Contribution rational if:

$$B(S_{-i} + 1) - B(S_{-i}) \geq c - s.$$

Sufficiently large s induces cooperation.

Structural Role

Collective action:

- Formalizes tension between private incentives and public welfare.
- Explains need for norms and enforcement.
- Connects institutional constraints to equilibrium selection.
- Links individual rationality to systemic outcomes.

It provides formal basis for governance design.

Structural Compression

Invariant

Public good problem arises when:

$$B(S + 1) - B(S) < c \quad \text{but} \quad n[B(S + 1) - B(S)] > c.$$

Mechanism

Externalities create divergence between individual and collective marginal incentives.

Cross-System Echo

- Economics: public goods and free-riding.

- Biology: cooperative behavior in populations.
 - Networks: shared infrastructure maintenance.
 - Control Theory: distributed optimization under local incentives.
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Concepts: collective-action, nash-equilibrium, institutions

Prerequisites: 8.2, 8.3

Enables: 8.5, 9.2

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