Constraint Architecture

Systems Thinking · Module 9 — Governance And Constraints

Node 9.1

Position in Knowledge Graph

This node formalizes governance as the design and composition of constraint layers over agent and institutional action spaces.

It builds on: - Institutions as Constraint Systems (8.1) - Institutional Stability (8.5)

It prepares for: - Principal–Agent Theory (9.2) - Hierarchical Structure (10.1)

Constraint architecture is the structural interface between institutions and governance.

Formal Statement / Definition / Construction

Let n agents have joint action space:

$$\mathcal{A} = \prod_{i=1}^{n} \mathcal{A}_i.$$

A **constraint layer** is a mapping:

$$C : \mathcal{A} \to \{0, 1\}.$$

The feasible set under C is:

$$\mathcal{A}^C = \{a \in \mathcal{A} : C(a) = 1\}.$$

A **constraint architecture** is an ordered tuple of constraint layers:

$$\mathbf{C} = (C_1, C_2, \ldots, C_m).$$

The induced feasible set is the intersection:

$$\mathcal{A}^{\mathbf{C}} = \bigcap_{k=1}^{m} \mathcal{A}^{C_k}.$$

Equivalently, define combined constraint:

$$C_{\mathbf{C}}(a) = \prod_{k=1}^{m} C_k(a),$$

so that:

$$\mathcal{A}^{\mathbf{C}} = \{a : C_{\mathbf{C}}(a) = 1\}.$$

Hard vs Soft Constraints

A hard constraint forbids actions by exclusion.

A **soft constraint** imposes penalties via modified utility:

$$\tilde{U}_i(a) = U_i(a) - \sum_{k=1}^{m} \pi_k(a),$$

where penalty functions satisfy $\pi_k(a) \geq 0$.

Soft constraints induce feasibility through incentive shaping, not outright exclusion.

Assumptions and Conditions

1. Each C_k is well-defined on $\mathcal{A}$.
2. Intersection is nonempty:
$$\mathcal{A}^{\mathbf{C}} \neq \emptyset.$$
3. Agents are constrained to select $a \in \mathcal{A}^{\mathbf{C}}$ for hard constraints, or respond to penalties for soft constraints.
4. Constraint layers are exogenous at this node (design dynamics deferred to later nodes).

No assumption is made about optimality of the architecture.

Proof or Mathematical Development

Feasible-Set Monotonicity Under Layering

Let:

$$\mathbf{C}_m = (C_1, \ldots, C_m), \quad \mathbf{C}_{m+1} = (C_1, \ldots, C_m, C_{m+1}).$$

Then:

$$\mathcal{A}^{\mathbf{C}_{m+1}} = \left(\bigcap_{k=1}^{m} \mathcal{A}^{C_k} \right) \cap \mathcal{A}^{C_{m+1}} \subseteq \bigcap_{k=1}^{m} \mathcal{A}^{C_k} = \mathcal{A}^{\mathbf{C}_m}.$$

Thus adding constraints can only restrict feasible action space.

This monotonicity is structural and does not depend on agent preferences.

Constraint Compatibility Condition

Constraint layers may conflict.

Compatibility requires:

$$\mathcal{A}^{\mathbf{C}} \neq \emptyset.$$

If:

$$\mathcal{A}^{C_i} \cap \mathcal{A}^{C_j} = \emptyset$$

for some i, j, then no action is feasible, and the architecture is infeasible.

Thus governance design must satisfy non-emptiness of feasible intersection.

Constraint-Induced Equilibrium Shift

Let a^* be equilibrium of unconstrained game.

If:

$$a^* \notin \mathcal{A}^{\mathbf{C}},$$

then a^* is infeasible.

Equilibrium must be computed on restricted domain $\mathcal{A}^{\mathbf{C}}$.

Thus constraints select equilibria by excluding regions of action space.

Structural Role

Constraint architecture:

- Provides the formal object for governance design.
- Explains how multiple institutional layers compose.
- Separates hard feasibility constraints from soft incentive constraints.
- Creates a lattice of feasible spaces ordered by inclusion.

This node is the gateway from institutions (rules) to governance (rule design).

Structural Compression

Invariant

Constraint architecture induces feasible set intersection:

$$\mathcal{A}^{\mathbf{C}} = \bigcap_{k=1}^{m} \mathcal{A}^{C_k}.$$

Mechanism

Layer composition restricts action space monotonically under inclusion.

Cross-System Echo

- Control Theory: admissible control sets and constraint tightening.
- Economics: legal constraints + incentive taxes as layered mechanisms.
- Networks: protocol stacks as constraint compositions.
- Biology: multi-layer regulatory constraints on gene expression.

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Concepts: governance, institutions, constrained-optimization, hierarchical-systems
Prerequisites: 8.1, 8.5
Enables: 9.2, 10.1

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