

Principal-Agent Theory

Systems Thinking · Module 9 — Governance And Constraints

Node 9.2

Position in Knowledge Graph

This node formalizes governance under asymmetric information and delegated action.

It builds on: - Collective Action (8.4) - Constraint Architecture (9.1)

It prepares for: - Incentive Compatibility (9.3) - Mechanism Design (9.4)

Principal-agent theory models how a governing entity designs incentives when it cannot directly observe actions.

Formal Statement / Definition / Construction

Let:

- Principal choose contract $w : \mathcal{Y} \rightarrow \mathbb{R}$,
- Agent choose effort $a \in \mathcal{A} \subseteq \mathbb{R}$,
- Output $y \in \mathcal{Y} \subseteq \mathbb{R}$.

Output is stochastic:

$$y = g(a) + \varepsilon,$$

where:

- $g : \mathcal{A} \rightarrow \mathbb{R}$ increasing,
- ε random noise with known distribution.

Agent utility:

$$U_A(a, w) = \mathbb{E}[w(y)] - c(a),$$

where $c'(a) > 0, c''(a) > 0$.

Principal utility:

$$U_P(a, w) = \mathbb{E}[y - w(y)].$$

Assumptions and Conditions

1. Principal cannot observe effort a directly (hidden action).
 2. Output y observable.
 3. Agent risk-neutral (unless specified otherwise).
 4. Contract enforceable.
 5. Agent has reservation utility $\bar{U}$.
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Proof or Mathematical Development

Agent Optimization (Incentive Constraint)

Agent chooses a to maximize:

$$\max_{a \in \mathcal{A}} \mathbb{E}[w(g(a) + \varepsilon)] - c(a).$$

First-order condition:

$$\frac{d}{da} \mathbb{E}[w(g(a) + \varepsilon)] = c'(a).$$

Let:

$$\mathbb{E}[w'(y)g'(a)] = c'(a).$$

This defines agent best-response $a^*(w)$.

Participation Constraint

Agent must receive at least reservation utility:

$$\mathbb{E}[w(y)] - c(a^*) \geq \bar{U}.$$

Principal Problem

Principal chooses $w(\cdot)$ to maximize:

$$\max_w \mathbb{E}[y - w(y)]$$

subject to:

1. Incentive constraint (IC),
 2. Participation constraint (PC).
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Second-Best Distortion

If effort observable, principal solves:

$$\max_a \mathbb{E}[g(a)] - c(a).$$

FOC:

$$g'(a) = c'(a).$$

Under hidden action, optimal a^* generally satisfies:

$$g'(a^*) < c'(a^*),$$

due to information rent required to satisfy IC.

Thus governance under asymmetric information induces distortion.

Structural Role

Principal-agent framework:

- Formalizes delegation under hidden action.
- Introduces incentive constraints.
- Connects institutional rules to contract design.
- Explains efficiency loss due to information asymmetry.

Governance becomes contract optimization under constraints.

Structural Compression

Invariant

Optimization under hidden action:

$$\max_w \mathbb{E}[y - w(y)] \quad \text{s.t. IC and PC.}$$

Mechanism

Information asymmetry introduces incentive compatibility constraint.

Cross-System Echo

- Control Theory: output-feedback control under partial observation.
 - Economics: employment contracts.
 - Networks: delegated computation with unverifiable actions.
 - Biology: signaling under costly effort.
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Concepts: principal-agent, incomplete-information, constrained-optimization

Prerequisites: 8.4, 9.1

Enables: 9.3, 9.4

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