# Incentive Compatibility

Systems Thinking · Module 9 — Governance And Constraints

Node 9.3

## Position in Knowledge Graph

This node formalizes incentive compatibility as a constraint ensuring truthful or desired behavior under strategic choice.

It builds on: - Principal–Agent Theory (9.2)

It prepares for: - Mechanism Design (9.4) - Hierarchical Governance (10.1)

Incentive compatibility is the core constraint in governance under private information.

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## Formal Statement / Definition / Construction

Let:

- Agents $i = 1, \dots, n$,
- Each agent has private type $\theta_i \in \Theta_i$,
- Mechanism defined by:
  - Allocation rule $x(\theta)$,
  - Transfer rule $t_i(\theta)$.

Let utility of agent $i$ be:

$$U_i(x, t_i; \theta_i).$$

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### Definition (Incentive Compatibility)

Mechanism is **incentive compatible** if truthful reporting is optimal.

Formally, for all $i$, for all $\theta_i, \hat{\theta}_i$:

$$U_i\big(x(\theta_i, \theta_{-i}), t_i(\theta_i, \theta_{-i}); \theta_i\big) \geq U_i\big(x(\hat{\theta}_i, \theta_{-i}), t_i(\hat{\theta}_i, \theta_{-i}); \theta_i\big).$$

This is dominant-strategy incentive compatibility (DSIC).

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### Bayesian Incentive Compatibility (BIC)

If types distributed with prior $p(\theta)$, mechanism is BIC if:

$$\mathbb{E}_{\theta_{-i}}\left[U_i(x(\theta_i, \theta_{-i}), t_i(\theta_i, \theta_{-i}); \theta_i)\right] \geq \mathbb{E}_{\theta_{-i}}\left[U_i(x(\hat{\theta}_i, \theta_{-i}), t_i(\hat{\theta}_i, \theta_{-i}); \theta_i)\right].$$

Expectation over others' types.

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## Assumptions and Conditions

1. Agents maximize expected utility.
2. Type space measurable.
3. Utility functions well-defined.
4. Mechanism enforceable.

No assumption of budget balance at this stage.

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## Proof or Mathematical Development

### Direct Revelation Principle (Statement)

For any equilibrium outcome of a mechanism with strategic reporting, there exists a direct mechanism that is incentive compatible and yields the same outcome.

Proof outline:

Given equilibrium strategy profile $s_i(\theta_i)$, define direct mechanism:

$$x^*(\theta) = x(s_1(\theta_1), \dots, s_n(\theta_n)),$$

$$t_i^*(\theta) = t_i(s_1(\theta_1), \dots, s_n(\theta_n)).$$

Truthful reporting replicates equilibrium outcome.

Full proof requires equilibrium refinement; detailed proof deferred to advanced mechanism design domain.

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### Incentive Constraint as Envelope Condition (Single-Agent Case)

For single agent with quasi-linear utility:

$$U(\theta) = v(x(\theta), \theta) - t(\theta),$$

IC implies monotonicity of allocation and envelope condition:

$$\frac{dU(\theta)}{d\theta} = \frac{\partial v(x(\theta), \theta)}{\partial \theta}.$$

Derivation assumes differentiability and single-crossing condition.

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## Structural Role

Incentive compatibility:

- Ensures alignment between private information and desired behavior.
- Converts hidden-type problems into constrained optimization.
- Central to governance under asymmetric information.
- Restricts feasible allocation rules.

It is the structural constraint underlying mechanism design.

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## Structural Compression

### Invariant

Truthful reporting optimal:

$$U_i(\theta_i) \geq U_i(\hat{\theta}_i).$$

### Mechanism

Incentive constraints restrict allocation and transfer functions.

### Cross-System Echo

- Control Theory: observer-based feedback under partial information.
- Economics: auctions and contract design.
- Networks: truthful routing protocols.
- Biology: signaling equilibria.

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*Systems Thinking · Module 9 — Governance And Constraints · Node 9.3*
**Concepts:** incentive-compatibility, mechanism-design, incomplete-information
**Prerequisites:** 9.2
**Enables:** 9.4, 10.1

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